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Extension in The Port Royal Logic[1]

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Abstract

This paper is a discussion of the meaning of *extension* in Arnauld and Nicole's *Logic or the Art of Thinking*. Contrary to the reading of Jean-Claude Pariente, who reads an idea's extension "intentionally" as the ideas defined in its terms, the paper defends a "referential" interpretation in which an idea's extension, although a set of ideas, tracks the objects that the idea signifies in the world. Pariente's reading makes truth a function of conceptual inclusion. The referential reading insures a correspondence theory of truth. It is argued that both readings account for essential truths, but only the referential reading accommodates the *Logic*'s commitment to contingent truth, sensation as part of the scientific method, the truth-conditions for categorical propositions, and the *Logic*'s account of false ideas and error. It is argued that, contrary to Pariente's reading, the subject terms of true affirmative categorical propositions carry existential import.

Keywords: *Port Royal Logic*, Extension, Arnauld, Pariente, false idea, contingent truth, existential import.

1 Introduction

The syllogistic proof theory of *The Port Royal Logic* (Antoine Arnauld and Pierre Nicole, *La Logique ou l'Art de Penser*, 1662) is unoriginal. It is a simple

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exposition of well know lore consisting of various syntactic markers and rules that identify the relations of the Square of Opposition and the valid moods. What is original, on the other hand, is its semantics. Due to its Cartesian dualism, the *Logic* rejects what had been the standard view that ideas stand for things because they instantiate in a "spiritual way" properties transferred by abstraction from the material substances they signify. The Cartesians, on the other hand, require a new account of signification and with it new truth-conditions for the categorical propositions of the Square. The key concept of the new semantics is extension.

The resulting theory looks superficially Aristotelian. Terms have extensions. It is assumed that the extension of a subject term is non-empty. A universal affirmative is true if the extension of the subject is contained in that of the predicate. A universal negative is true if the two extensions are disjoint. A particular affirmative is true if the intersection of the two extensions is non-empty. A particular negative is true if the extension of the subject apart from that of the predicate in non-empty. The standard relations of the Square then follow. A and E propositions are contraries; I and O subcontraries; I is subaltern to A and O to E; A is the contradictory of O and E of I. Barbara, Calerent and the rest are all valid.

What is not evident here is the meaning of extension. What do terms signify, and how are these *significata* grouped into extensions? The standard interpretation in the Middle Ages was Aristotelian and "extensional." Terms were understood to be "said of" or signify material substances "outside the mind." Leibniz, who came after Arnauld and Nicole, was famous for allowing the terms in his syllogistic to be interpreted systematically as signifying either intensions (spiritual ideas) or extensions (material substances). A problem facing a theory like Leibniz's, however, is that truth appears to be a matter of intentional relations alone, of idea containment and exclusion. Lost, it seems, is a robust correspondence theory of truth — the common sense core of Aristotelian semantics — in which propositions describe what happens outside the mind.

Arnauld and Nicole have it both ways. They give an intentional interpretation to extensions, and hence to the truth-conditions of the Square. At the same time they understand signification "referentially" as a relation that stands between mental terms and material things. On their account extensions are made up of ideas, but they nevertheless track the inclusion and exclusion relations of material substances. The result it a robust correspondence theory of truth. This paper explores the meaning of extension that makes this possible.

2 The Definition of Extension

The term *extension* is used for the first time in the *Logic* as a technical term to stand for the semantic value of terms. Although the term *extensio* (*étendue* in French) had a long history in natural philosophy and continued to be used by the Cartesians for the essence of matter, its quite different use as a technical term in semantics was new to the *Logic*, but it is not entirely new. In the *Logic* a term's extension is what a proposition quantifies over, and it is made up of ideas rather than things. That we quantify over ideas is not as hard to motivate as it might appear. In Aristotle's terminology a term's extension is what it is "said of." There is some reason, moreover, to think that Aristotle may have intended the subject term of a categorical proposition to range not over things, but over the species subordinate to the term. For example, Aristotle cites as cases confirming *some animals are viviparous* not individuals but the species *man*, *horse*, and *camel*.[2] For the *Logic*'s authors, like many medieval nominalists, species are ideas. Some medieval commentators explain, moreover, this relation in terms of cognates of *extension*. When Duns Scotus comments on the meaning of the *Isagoge* when Porphyry says that a genus is "more of a collection" than its species,[3] Scotus uses the verb *extendere*. A genus, he explains, is "more [*maior*] a universal" because it is extended [*exdenditur*] to more things [*plura*].[4] When Cajetan and Toletus comment on this passage, they say that "more" is used "extensively" [*extensive*].[5]

[2]See Thompson, Michael (1995). 'The representation of life'. In: Gavin Laurence, R. H., Warren Quinn (ed.) *Phillipa Foot and Moral Theory*. Oxford: Clarendon Press. pp. 247–296.

[3]Descendentibus igitur ad specialissima necesse est diuidentem per multitudinem ire, ascendentibus uero ad generalissima necesse est colligere multitudinem (collectiuum enim multorum in unam naturam species est, et magis id quod genus est, particularia uero et singularia semper in multitudinem e contrario diuidunt quod unum est; participatione enim speciei plures homines unus, particularibus autem unus et communis plures; diuisiuum enim est semper quod singulare est, collectiuum autem et adunatiuum quod commune est). De speciem 12, *Isagoge a Boethio translata* [02] II.

[4]Ad aliud dico quod genus non est magis uniuersale, quia "magis" dicit intensionem formae eius cui adiungitur, sed quodammodo maius uniuersale, quia extenditur ad plura, sicut quaternarius est maior numerus binario, non magis. Sicut etiam una species specialissima non dicitur magis species quam alia, licet habeat plura contenta sub se. III, 7-8.23, p. 40. John Duns Scotus (1999). 'Quæstiones in librum porphyrii isogoge'. In: Girard J.Etzkorn, et al. (ed.) *B. Iohannis duns scoti opera philosophica*,. St Bonaventure, N.Y.: Franciscan Institute.

[5]Ad hoc breviter dicitur, quod esse magis collectivum multorum potest intelligi dupliciter. Uno modo intensive; et sic species est magis collectiva, quia magis unit adunata, ut ratio adducta probat. Alio modo extensive; et sic genus est magis collectivum, quia multo plura sub sua adunatione cadunt, quam sub specie ambitu. Unde species et genus se habent sicut duo duces, quorum alter habet exercitum parvum, sed valde unanimem, alter exercitum magum, sed diversam factionum. Porphyrius autem loquebatur hic de extensiva collectione, et ideo dixit genus est magis collectivum. P. 56, Caietanus, Thomas De Vio (1936). 'Commentaria

The interpretation of *extension* in the context of the *Logic*'s semantic theory is, however, a matter of contention. The problem is that extension is defined in terms of more basic concept of "inferiority," an undefined relation among ideas:[6]

I call the *extension* of an idea the subjects to which this idea applies. These are also called the inferiors of a general term, which is superior with respect to them. For example, the idea of a triangle in general extends to all the different species of triangles.

The background theory assumes that language is mental and that its terms are ideas. This definition of an idea's extension specifies that it is made up of all the subjects" "inferior" to it. It is clear what subjects are. These are the ideas that may occupy the subject position in a categorical proposition. The inferiority relation, however, is problematic. Kneale and Kneale highlight the difficulty as follows:

> . . . according to Arnauld and Nicole, the extension of a general term is the set of its inferiors, but it is not clear whether the inferiors of which they speak are supposed to be species or individuals. When working out their example they say that the idea of triangle in general extends (*s'étend*) to all the various species of triangle, but in the next paragraph they make the point that the extension of a term, unlike its comprehension, might be cut down without destruction of the idea ('*on peut la reserrer quant à son étendue . . . sans que pour cela on la détruise'*), and this is not true of the set of species falling under a genus. . . . The confusion of their exposition seems to be due to their use of the word 'inferiors', which is itself metaphorical and unclear. It will be remembered that in medieval representations of

in porphyrii isagogen, ad praedicamenta aristotelis scripta philosophica'. In: Isnardus M. Marega, O.P. (ed.). Roma: Insitutum Angelicum.

Hic notandum est dupliciter aliquid posse dici magis collectiuum; ut notat Cajetanus: priori modo id quod est magis unum, quod dicitur magi collectiuum intensive; altero modo id, quod plura comprehendit, & sic dicitur magis collectiuum extensive: Iuxta hoc intellige minus universale esse magis collectiuum intensive; quia magis sunt unum quae in minus universali conveniunt quam que solum in magis universali: at magis universale est magis collectiuum extensive quia sub se plura continet; & sic loquitur Porphyrius. Caput II, p. 53, Toletus, Franciscus (1985 [Köln 1615/16]). 'Cur de specie post genus & non de differentia egerit?'. In: Risse, W. (ed.) *Commentaria in universam aristotlis logicam. Opera omina philsophica.* Hildesheim: Gerg Olms. pp..

For a fuller discussion see Martin, John N. (2012). 'Existential commitment and the cartesian semantics of the port royal logic'. In: Beziau, J.-Y. (ed.) *New perspectives on the square of opposition.* Peter Lang. pp. 61-97..

[6] J'appelle étendue de l'idée, les sujets à qui cette idée convient, ce qu'on appelle aussi les inferieurs d'un terme général, qui à leur égard est appellé supérieur, comme l'idée du triangle en général s'étend à toutes les diverse espéces de triangles. (*LAP* I:6, *KM*:V 145, *B* 40.)

> Porphyry's tree individuals such as Socrates, Plato, and Brunellus were often mentioned at the bottom of the table in which all the other entries were general terms.[7]

Because the text does not define the inferiority relation, its meaning must be gathered indirectly from previous usage, its role in the wider theory, and occasional examples. There are two contending interpretations of inferiority and hence of extension. To explain them, it is necessary to first sketch some background theory.

Basic to the *Logic*'s semantic theory is the notion of comprehension. As a matter of Providence every idea possesses an intentional content, which the *Logic* calls its *comprehension*. This consists of a series of modes. In modern terms we would call it a set. It is this set of modes that determines what the idea refers to or, in the language of its time, what it "signifies:" an idea *signifies* all those actual entities that satisfy all the modes in its comprehension. Comprehension is a Cartesian version of the medieval notion of objective being. According to this doctrine an idea is a mode of the soul, and thus in a broad sense part of the soul's "form." For this reason, an idea is said to have *formal being*. Because the idea is also related to modes that determine the objects it signifies, the idea is also said to have *objective being*.[8]

Comprehension not only determines what an idea signifies. It also provides its identity conditions. Idea A is identical to idea B if, and only if, A and B have the same comprehension. Comprehension also explains the mental operations of abstraction and restriction. Abstraction is the operation on ideas that forms from an idea or perception A a new, more abstract idea B by removing modes from the comprehension of A. Restriction is the operation that forms a new idea A from ideas B and C by assigning to it the comprehension that consist of the modes shared by the comprehensions of B and C. Comprehension also defines the relation of idea containment. Idea A *contains* idea B if, and only if, every mode in the comprehensions of B is in the comprehension of A. With this background it is now possible to explain the alternative interpretations of the inferiority relation and hence of extension.

[7]Kneale, Kneale William and Marth (1962). *The development of logic*, Oxford, Clarendon Press., 318–320.

[8]The "logic of terms" described here is described in Book I. The central notions, which are sketched in this paragraph, are introduced in I:vi.

3 The Intentional Interpretation

What may be called *the intentional interpretation* is developed by Jean-Claude Pariente.[9] It stresses that extensions consists of ideas. The interpretation has the support of previous usage. It draws on the usage in in medieval logic in which the species of a genus are referred to as its "inferiors." Accordingly, he identifies the inferiority relation with the species to genus relation. Because a species is defined in terms of its genus, the *Logic* holds that the comprehension of a species includes the modes that define its genus. On this reading, then, comprehension inclusion determines idea containment, and the inferiority relation is simply the converse of the idea containment relation: idea A *is inferior* to idea B if, and only if, idea B is contained in idea A. In other words, the view holds that the extension of an idea consists of all the ideas that are defined in its terms.

An immediate consequence of the interpretation is that truth is conceptual and determined entirely by comprehensions. This implication follows from the *Logic*'s truth-conditions for categorical propositions, which are formulated in terms of extension. A universal affirmative *every* S *is* P is true if, and only if, the extension of S restricted by that of P is the same as that of S. Equivalently, *every* S *is* P is true if, and only if, the extension of S is a subset of that of P. The interpretation, then, has the consequence that *every* S *is* P is true if, and only if, the comprehension of P is a subset of that of S.

The interpretation has a number of considerations in its favor. We have already mentioned that it was common in medieval logic to refer to the species' under a genus as its inferiors.[10] It is also true that, as in the definition of extension quoted above, the examples the *Logic* gives of inferior ideas are often species of a genus. A more important argument in its favor is that it supports

[9] Pariente, Jean-Claude (1985). *L'analyse du language à Port-Royal*, Paris, C.N.R.S. Éditions de Minuit., Chapter 8, 227–258.

[10] Here are two examples of *inferior* used to refer to the relation of species to its genus from John Buridan's *Summulae* (Buridan, John (2001). *Summulae de dialectica*, New Haven, Yale University Press.):

> 3.1.5 ... *Tunc ergo sequitur quod omnis terminus universalis dicitur de subjecto, quia habet sub se terminum inferiorem, de quo est praedicabilis essentialiter,*
> And then it follows that every universal term is said of a subject, because it has an inferior term under it of which it can be predicates essentially. K 149.
> 3.2.2. ... *Quia locutus est saepe Philosophus de dici de subjecto ... scilicet ... inter ea quae praedicantur essentialiter de suis inferioribus, quae diximus dici de subjecto,*
> But the philosopher has frequently spoken about beings said of a subject ... namely ... those that are predicated of its inferiors, which we called said of a subject. K 155. Author's translation.

the *Logic*'s doctrine of scientific knowledge.

The *Logic* takes over a version of Descartes' view that scientific knowledge rests, in the *Logic*'s terminology, on the soul's understanding of the comprehensions of ideas clearly and distinctly:[11]

> . . . [Descartes] understands the word 'idea' in the proposition that 'everything that I perceive clearly as being in the idea of a thing can correctly be asserted of that thing' [*tout que je vois clairement être enfermé dans l'idée d'une chose, eut avec vérité être affirmé de cette chose*], which he claims, with good reason, to be the foundation of all the natural sciences. If, examining the idea that I have of a triangle (by reflecting on the perception that I have of it), I find that the equality of its three angles to two right angles is contained in [*est enfermé dans*] this idea or perception, I can correctly assert that every triangle has three angles equal to two right angles[12]

The *Logic* codifies this doctrine by incorporating it into its first axiom of rational inquiry:[13]

> *Everything contained in the clear and distinct idea of a thing can be truthfully affirmed of it.*

Propositions that affirm the content of an idea's comprehension are also known as essential truths, and these are described as universal and necessary, both properties of scientific knowledge.[14] Real definitions count as essential truths even though they are not always obvious and may need to be proven from

[11] Descartes holds that the truths of logic and mathematics are true, necessary and eternal because God wills them to be so. Response to 6th set of Objections, VI: HR II, 238; Pléiade 535, AT 7, 431–433. Moreover, in *Meditation* V he says such truths are about immutable nature and as such may be true even if their subject term is an idea that fails to stand for something that actually exists:

> The most important point is that I find in myself countless ideas of things that can't be called *nothing*, even if they don't exist anywhere outside me. For although I am free to think of these ideas or not, as I choose, *I didn't invent them*: they have their own true and immutable natures, which are not under my control. Even if there are not and never were any triangles outside my thought, still, when I imagine a triangle I am constrained in how I do this, because there is a determinate nature or essence or form of *triangle* that is eternal, unchanging, and independent of my mind.

Meditation V.05, AT 7.64, 76–77. English translations of the *Meditations* are from Descartes, René (2007-2010). *Meditations on first philosophy*, MS.

[12] *VFI* Chapt. 6: *KM*:I 206; *G* 73.

[13] *LAP* IV:7, *KM*:V 381–382, *B* 250.

[14] *LAP* IV:13, *KM*:V 398, *B* 263.

what is already known.[15] Because essential truths like real definitions are often universal affirmatives with a more general term as predicate, they are said to describe the "causes" of things. The genus, for example, is the cause of the species.[16]

On the intentional interpretation, then, truth and science are entirely a matter of ideas. A universal affirmative is true if the content of the predicate is included in that of the subject, and science consists of understanding clearly and distinctly that the modes expressed by the predicate are expressed by the subject. Although there happens to be a world outside the mind and the terms of language signify it, truth and knowledge are explained totally without reference to it.

4 The Referential Interpretation

The inferiority relation's alternative reading presents quite a different picture of the purpose of language. On this reading, which we shall call *the referential interpretation*, inferiority is defined in terms of signification, and even though extensions are sets of ideas, they map one-to-one to sets of objects outside the mind. They do so, moreover, in such a way that when the extension of a subject is a subset of the extension of a predicate, the objects signified by the subject are simultaneously a subset of the objects signified by the predicate. On this reading, then, instead of defending a completely closed version of idealism, the *Logic* espouses a robust correspondence theory of truth, and in doing so, falls in the tradition of earlier logic.

The interpretation's key definitions are most clearly stated algebraically. Let us posit undefined sets of *ideas*, *modes*, and (*actual*) *substances*, both material and spiritual. It is assumed that the actual world consists of substances in which modes inhere,[17] and that there is a 1-1 onto mapping from ideas to set of modes called *comprehension-sets*. Signification is defined in terms of comprehension, and extension in terms of comprehension: idea A *signifies* X if, and only if, all the modes in the comprehension of A are instantiated in

[15] *LAP* I:12, *KM*:V 170–174, *B* 60–63.

[16] *LAP* IV:6, *KM*:V 380, *B* 249. In one edition of the *Logic* the necessity of scientific knowledge is highlighted by the remark that if a scientific proposition is possibly true, it is necessary, or in modal logic: $(\Box P \vee \Box{\sim}P) \wedge \Diamond P \vDash \Box P$:

> Thus when a geometer conceives that a line could be described by four or five different motions, he never took the trouble to draw the line, because it was enough for it to be possible in order for him to consider it as true.

LAP IV:13, *KM*:V 398, *B* 263.

[17] The *Logic* also allows for second intention (modes that inhere in modes), but these are irrelevant here.

X; *the extension* of idea A is the set of all ideas B such that, for any X, if B signifies X, then A signifies X. It follows that the extension of A is the set of all B such that for any X, if all the modes in the comprehension of A are true of X, then all the modes in the comprehension of B are true of X. Although the concept does not appear in the *Logic*, it is useful to group together the *significata* of an idea. Let us call the *significance range* of idea A the set of all X that A signifies.[18] It follows that the extension of idea A is the set of all ideas B such that the significance range of B is a subset of that of A. A series of algebraic relations follow. The structure of ideas ordered by containment is isomorphic to that of comprehensions ordered by set-inclusion. There is an antitonic homomorphism from comprehension-sets ordered by set inclusion (and hence also from ideas ordered by containment) to the family of significance ranges ordered by set inclusion. Hence, ideas and comprehensions are dual to significance ranges in the algebraic sense of duality. Lastly, there is an onto homomorphism from the family of significance ranges to the family of extensions ordered by set inclusion. It follows that, like significance ranges, ideas and comprehension-sets are dual to extensions.[19]

The important consequence of these algebraic gymnastics is that an idea's extension, which is made up of ideas, tracks its significance range, which is made up of things in the world. The extension of one idea is included in a second exactly when the significance range of the first is included in that of the second. As a result, the truth of a universal affirmative, which is a matter of extensional inclusion, corresponds to the subordination of sets of things outside the mind. The result is a correspondence theory of truth.

Although the reading defends a correspondence theory, it should be noted that it also preserves "essential truths." If a universal affirmative is true according to the intentional interpretation, then it is also true according to the referential interpretation. This result follows because if the comprehension of P is a subset of that of S, then whatever S signifies so does P, and hence the extension of S is a subset of that of P. Thus, if the comprehension of P is included in that of S, *every S is P* will continue to be true under the referential interpretation. What distinguishes the referential from the intentional interpretation is rather its treatment of non-essential truths, which often hold

[18] A term's significance range is what Leibniz and modern logicians like Carnap call its extension.

[19] For a more detailed discussion of these algebraic relations with some proofs see Martin, John N. (2016b). "The structure of ideas in the Port Royal logic", *The Journal of Applied Logic* 19, pp. 1–19. In some of his logic papers Leibniz held that there was an inverse 1-1 onto antitonic mapping from significance ranges (which he called a term's extension) to ideas, and hence that the inverse duality also holds. See Lenzen, Wolfgang (2004). 'Leibniz's logic'. In: D. M. Gabbay, J. Woods (ed.) *Handbook of the history of logic*, vol. 3. Amsterdam: Elsevier-North Holland. pp. 1–83., p. 15.

even when the comprehension of the predicate is not included in that of the subject. The *Logic*, in fact, has a great deal to say about the semantics of contingent truth, the possibility of which is not even allowed for on the intentional reading.

5 Contingent Truth, Accidents, Non-Essential Divisions, and Factitious Ideas

The *Logic* is explicit in maintaining that there are non-necessary, non-essential contingent truths:

> The first reflection is that it is necessary to draw a sharp distinction between two sorts of truths. First are truths that concern merely the nature of things and their immutable essence, independently of their existence. The others concern existing things, especially human and contingent events, which may or may not come to exist when it is a question of the past. I am referring in this context to the proximate causes of things, in abstraction from their immutable order in God's providence, because on the one hand, God's providence does not preclude contingency, and on the other, since we know nothing about it [i.e. contingent creation], it contributes nothing to our beliefs about things. For the other kind of truth [viz. of essential natures], since everything [of this sort] is necessary, nothing is true that is not universally true. So we ought to conclude that something is false if it is false in a single case.[20]

Examples of contingent propositions include:[21]

> *The king of China has converted to Christianity.*
> *Constantine was baptized by St. Sylvester.*
> *St. Peter was in Rome.*

The authors also accept the five "predicables" of traditional logic, which include accidents. An accident is a mode "that is in no way necessarily connected to the idea of a thing, so that one can easily conceive the thing without

[20] *LAP* IV:13, *KM*:V 398, *B* 263. In the first edition of 1662 the text continues:

> On the contrary, possibility [i.e. even a single possible instance, a *possibilium*] is a sure mark of the truth with respect to what is recognized as possible, whenever it is a question only of the essence of things. For the mind cannot conceive anything [concerning essences] as possible unless it conceives it as true according to its existence.

[21] *LAP* IV:13, *KM*:V 398–400, *B* 263-265.

conceiving the mode." An example is prudence.[22] Consider the proposition *Peter is prudent*, which is a universal affirmative according to the *Logic*. If it is true, then according to the theory of truth it is because the extension of *Peter* is a subset of that of *prudent*. On the other hand, because prudence is an accident, it is not the case that the comprehension of *prudent* is included in that of *Peter*. Hence extensional inclusion is independent of comprehension inclusion.

The intentional interpretation would be more plausible if the terms of language consisted only of genera and species. The *Logic*, however, entertains a rich variety of non-species terms in propositions that describe facts but are not real definitions. These include not just adjectives describing accidents, but also various common nouns abstracted from perception, and many complex nouns formed by restriction, including accidental divisions.[23]

6 Sensation and Analysis

Within the *Logic*'s epistemology, sensation has an important role in the justification of knowledge of contingent truths. The mechanism is explained in *On True and False Ideas*.[24] There Arnauld makes clear that he rejects Malebranche's version of representational realism. He rejects the view that the mind apprehends an intermediary or representation — Malebranche's ideas — that stands between the soul and the object of sensation. Arnauld, however, is not a simple direct realist. His view is that during sensation God instantiates in the mind a mental mode, called a perception. This mode, liked an idea, is a mental act that has both formal and objective being. It has formal being because it is a mode of the soul. It has objective being because like an idea it has a content, which is a series of modes. If the perception is veridical, these modes are true of the object of which it is a perception. The soul is both aware of these modes and is self-aware. It is conscious that the perception is occurring and that the perception has the content it does. Simultaneously, in the normal case, God causes it to happen that the sensation is veridical. He causes it to be the case that the object of sensation is present outside the mind, that it is causally affecting the body's organs of sensation, and that the object possesses those properties of extended substances that are contained in the content of the soul's perception. The doctrine is not perceptual representationalism be-

[22] *LAP* I:7, *KM*:V 150, *B* 43–44.

[23] On non-species ideas of various sorts including those formed by abstraction and restriction see Martin, John N. (2016b). "The structure of ideas in the Port Royal logic", *The Journal of Applied Logic* 19, pp. 1–19. For accidental non-species ideas used in "division" see *LAP* II:15, *KM*:V 243–246, *B* 123–125.

[24] *VFI*:5–6,28; *KMI*:201–205,357–8; *G* 69–72,216.

cause the soul is not aware of representations. Rather, it is directly aware of the modes instantiated in objects outside the mind. It is not simple direct perception, however, because the soul is also self-reflective. It is aware that it is experiencing a perception.

What is important for the purposes of this paper is that sensation has a role in justifying the knowledge of contingent truths. The authors hold that, as a general rule, when the soul has a sensation of S as P, the proposition *S is P* is true. They go so far as to state as one of the axioms of scientific knowledge that received opinion grounded in sensation is well justified:

> When the facts that the senses can easily judge are witnessed by a great number of persons from different times, different nations, and diverse interests, who speak about them as if from personal experience, and who cannot be suspected of having conspired to maintain a lie, they should be considered as constant and indubitable as if we had seen them with our own eyes.[25]

In support the authors offer as a demonstration of this axiom a version of Descartes' argument based on the premise, confirmed by the idea of God, if not accepted on faith, that God is not a deceiver. It follows that the proposition *I exist* is true, which, it should be stressed, here is a contingent truth.

In *On True and False Ideas* Arnauld also gives a version of Descartes' demonstration justifying contingent knowledge: if my material body and other people do not exist outside the mind, then God is a deceiver; but God is not a deceiver; therefore my material body and other people do exist outside the mind.[26] It follows that sensations of material modes are in general veridical. For example, sensations of pain correspond to motions of the body, and sensations of color and touch correspond to motions in the world.[27]

Because universal generalization from a particular is invalid, the *Logic*'s authors reject induction.[28] They nevertheless advocate a "method of discovery" called *analysis* or *resolution* for identifying the causes of individuals or species. Here the sense of "cause" is drawn from the Platonic and Aristotelian tradition, which holds that, in a sense, a genus is the cause of its species.[29]

The paradigm the authors have in mind is a chain of reasoning in syllogisms in the mood Barbara, which may be reconstructed as follows. Each syllogism in the chain has as a minor premise that affirms a species of a particular individual, "the effect." Its major premise is a universal affirmative predicating

[25] Axiom 11, *LAP* IV:7, *KM*:V 382, *B* 251.
[26] *VFI*:28, *KMI*:355, *G* 213–214.
[27] *VFI*:28, *KMI*:357–358, *G* 216–217.
[28] *LAP* IV:6, *KMV*:377, *B* 247.
[29] *LAP* IV:2, *KMV*:362–366, *B* 233–237.

a genus of the species. The syllogism's conclusion affirms that the particular effect falls under the genus, which is its "cause." In each syllogism in the series the major premise consists of a classification typical of scientific knowledge. The minor premise has as its subject the effect, which is the individual term derived from sensation. It affirms of it a mode either that is directly sensed in the case of first syllogism of the series, or a mode established by the immediately previous syllogism if it is a syllogism later in the series. Each subsequent syllogism establishes that the effect falls under a more abstract cause than the one prior in the series. The final syllogism establishes that the effect is caused by its highest genus. For example,

> *Socrates is a human. Every human is an animal.* ∴ *Socrates is an animal.*
> *Socrates is an animal. Every animal is a living creature.* ∴ *Socrates is a living creature.*
> *Socrates is a living creature. Every living creature is a body.* ∴ *Socrates is a body.*
> *Socrates is a body. Every body is a substance.* ∴ *Socrates is a substance.*

The perception of the individual Socrates includes the modes *rational*, *self-moving*, *living*, *corporeal*, and *being*, and these become included in the comprehension of the idea *human* abstracted from the perception. The subject of the initial minor premise is an individual term, in this case the proper name *Socrates*, and the premise as a whole is a report of knowledge drawn from sensation. Each syllogism has a major premise affirming a real definition, which are examples of scientific knowledge. The conclusion of each syllogism reports further knowledge about the individual. The chain of reasoning supporting it is based in part on sensation and in part on science.

This example is contrived to illustrate the standard case in which scientific knowledge, which is represented by the major premises, predominates, and in which contingent knowledge, which is represented by the minor premise of the first syllogism, plays a lesser role. The example of analysis the authors actually provide, however, highlights the importance of sensory knowledge and contingent propositions. The sense of cause in the example is that in which a child is caused by ("is the descendant of") his or her parent. The analysis shows that an individual is the nth. generational descendant of St. Louis. Let us call this individual S_n and St. Louis S_0. The sequence of Barbaras makes use of intermediate premises S_i *is the descendant of* S_{i-1}. The "analysis" consists of the series:

> S_{n-1} *is the descendant of* S_{n-2} , S_n *is the descendant of* S_{n-1}, /∴. S_n *is the descendant of* S_{n-2};

S_{n-2} is the descendant of S_{n-3}, S_n is the descendant of S_{n-2}, /∴ S_n is the descendant of S_{n-3};
. . . ;
S_1 is the descendant of S_0, S_n is the descendant of S_1 /∴ S_n is the descendant of S_0.[30]

7 Truth-Conditions for Categorical Propositions

A further difficulty for the intentional interpretation of extension shows that the *Logic*'s authors had in mind propositions about the actual world rather than just necessary definitional relations among ideas. It is clear from the axioms that lay out the truth-conditions for categorical propositions in Book II that they intended relations of inclusion and exclusion among extensions to correspond to inclusion and exclusion relations among significance ranges. Here it will be sufficient to discuss the truth-conditions for the universal negative.[31] As Sylvan Auroux has pointed out, the intentional interpretation of extension yields incorrect truth-conditions.[32] The truth-conditions for a universal negative are stated in terms of extension: *no S is P* is true if, and only if, the extensions of S and P have no idea in common or, in set theoretic terms, the intersection of their extensions is empty. On the intentional interpretation the extension of idea A is the set of all ideas B such that all the modes in the comprehension of A are modes in the comprehension of B. But the truth of a universal negative is not a function of idea-definition but of facts outside the mind. Suppose that *no doctor is a thief* is true but *no doctor is a poet* is false. It is not because there is no idea *doctor-thief* but there is an idea *doctor-poet*. Both ideas can be made by restriction. On the intentional interpretation the

[30] IV:2, *KMV*:367, *B* 238. Hobbes had a similar account, and Leibniz advanced more formalized versions of analysis or resolution in various logic papers. In his formal logic he represents a categorical predicate as a series $P_1 \dots P_n$ of concatenated terms which are intended to display in the syntax what is called in the *Logic* the term's comprehension. He then employs an inference rule S *is* $P_1 \dots P_n \vdash S$ *is* $P_1 \dots P_{n-1}$. See, for example, *De arte combinatoria* in Parkinson, G. H. R. (1966). *Leibniz, logical papers*, Cambridge, Clarendon Press., and Swoyer, Chris (1995). "Leibniz on intension and extension", *Nous* 29, pp. 96–114. Hobbes' account of analysis and synthesis is very similar to Arnauld and Nicole's. See Hobbs, Thomas (1991 [1839-1845]). Collected works of Thomas Hobbs. In: Molesworthy, W. (ed.). London: Routledge, Thommes Press. *Concerning Body* [*De corpore*] I.6.1 Section 4.

[31] For a full discussion of the truth-conditions of all four categorical propositions and their referential interpretation as a development of medieval characterizations of the four propositional types in terms of characteristic ascents and descents to singular propositions see Martin, John N. (2013). "Distributive terms, truth, and the Port Royal logic", *History and Philosophy of Logic*, pp. 133–154, and Martin, John N. (2016a). "A note on 'Distributive terms, truth, and the Port Royal logic' ", *History and Philosophy of Logic* 37:4, pp. 391–392.

[32] Auroux, Sylvain (1993). La logique des idées, Montréal, Paris, Bellarmin, Vrin., 135.

one would both be in the extensions of *doctor* and *thief*, and other in those of *doctor* and *poet*. What makes the one proposition true but the other false, however, is not ideas defined in their terms but rather facts outside the mind. The significance range of *doctor-poet*, which is the intersection of the ranges of *doctor* and *poet*, is non-empty. On the other hand, that of *doctor-thief* is empty, and this is the intersection of those of *doctor* and *thief*.

8 False Ideas and Error

A further consideration in support of the referential interpretation is that it is presupposed in the *Logic*'s doctrine of false ideas and their role in the explanation of error. The notion that a false affirmation *every S is P* can generate a false idea *something that is S is P* was part of medieval logical lore. Descartes refers to such ideas in the *Meditations* as those that possess "a certain material falsity [*falsitas materialis*], which arises when they represent something non-real as if it were a real thing [*cum non rem tanquam rem repraesentant*]."[33] In III.6 he discusses the ideas *goat* and *chimera* in a context that suggests he thinks *goat* would normally be what we take to be a true idea, on the one hand, and *chimera* to be a false idea, on the other. The *Logic* explains the doctrine as follows:

> If the objects represented by these ideas, whether of substances or modes, are in fact such as they are represented to us, one calls them

[33]Descartes explains a false idea as follows:

> 19. . . . cætera autem, ut lumen & colores, soni, odores, sapores, calor & frigus, aliacque tactiles qualitates, nonnisi valde confuse & obscure a me cogitantur, adeo ut etiam ignorem an sint verae, vel falsæ, hoc est, an ideæ, quas de illis habeo, sint rerum quarundam ideæ, an non rerum. Quamvis enim falsitatem proprie dictam, sive formalem, nonnisi in judiciis posse reperiri paulo ante notaverim, est tamen profecto quaedam alia falsitas materialis in ideis, cùm non rem tanquam rem repræsentant : ita, exempli causâ, ideæ quas habeo caloris & frigoris, tam parum clarae [44] & distinctae sunt, ut ab iis discere non possim, an frigus sit tantùm privatio caloris, vel calor privatio frigoris, vel utrumque sit realis qualitas, vel neutrum. Et quia nullæ ideæ nisi tanquam rerum esse possunt, siquidem verum sit frigus nihil aliud esse quàm privationem caloris, idea quæ mihi illud tanquam reale quid & positivum repræsentat, non immerito falsa dicetur, & sic de cacteris.
>
> 20. Quibus profecto non est necesse ut aliquem authorem a me diversum assignem ; nam, si quidem sint falsæ, hoc est nullas res repraesentent, lumine naturali notum mihi est illas a nihilo procedere, hoc est, non aliam ob causam in me esse quàm quia deest aliquid naturæ meæ, nec est plane perfecta ; si autem sint veræ, quia tamen tam parum realitatis mihi exhibent, ut ne quidem illud a non re possim distinguere, non video cur a me ipso esse non possint.

Meditations III.19-20; *AT* 7.43, 45–47.

> true. If they are not such, then, in the manner in which they could be, they are false. [*si ils ne sont pas tels elles sont fausses en la maniere qu'elles les peuvent être*], and this is what one calls in the schools beings of reason, which consist ordinarily of the assemblage that the soul makes out of two ideas real in themselves, but which are not joined in truth to form a single idea, as that which one can form of a mountain of gold is a being of reason because it is composed of two ideas, of mountain and of gold, which it represents as unified though they really would not be so.[34]

Accordingly, a true idea represents things as they are. A false idea is a factitious idea that does not represent things as they are. In more technical terms, a false idea is one the comprehension of which contains modes that are not jointly true of any actually existing things. Some like *golden mountain* express modes that not even possibly jointly true of anything.[35]

Both Descartes and the authors of the *Logic* assign to false idea a central role in their explanation of error and the moral failings resulting from entails. Descartes holds that, "

> . . . the chief and most ordinary error that arises in them consists in judging that the ideas which are in us are like or conformed to the things that are external to us . . . .[36]

The *Logic* offers a psychological account. As children we habitually make a false judgement *S is P*, and as a result combine its terms to make a new complex idea *an S that is P*, which fails to signify anything real. Any affirmation we

[34] *LAP* I,2, *KM* V,136, *B* 32:

> Que si les objets représentés par ces idées, soit de substances, soit des modes, sont en effet tels qu'ils nous sont représentés, on les appelle *véritables* : que si ils ne sont pas tels elles sont *fausses* en la manière qu'elles les peuvent être; & c'est ce qu'on appelle dans l'école *êtres de raison*, qui consistent ordinairement dans l'assemblage que l'esprit fait de deux idées réelles en soit, même qui ne sont pas jointes dans la vérité pour en former une même idée, comme celle qu'on se peut former d'une montagne d'or, est un être de raison, parce qu'elle est composées des deux idées de montagne & d'or, qu'elle représente comme unies, quoiqu'elles ne le soient point véritablement.

[35] *LAP* I,ii, *KM* V,136, *B* 32.

[36] *Meditations* III.6. *AT* 7.37, 37:

> *Præcipuus autem error et frequemtissimus qui possit in illis reperiri, consistit in eo quod ideas, quæ in me sunt, iudicem rebus quibusdam extra me positis similes esse siue conformes. . .*

Literally the text reads: "consist in this that I might judge that ideas, which are in me, are similar to things posited as external to me but without conforming [to them]."

later make using the false idea as a subject term cannot be true because its subject term lacks signification:

> Because we were children before we became adults, and because external things acted on us, causing various sensations in the soul by the impressions they made on the body, the soul saw that these sensations were not caused in it at will, but only on the occasion of certain bodies, for example, when it senses heat in approaching the fire. But it was not content to judge merely that there was something outside it that caused its sensations, in which case it would not have been mistaken. It went further, believing that what was in these objects was exactly like the sensations or ideas it had on these *occasions*. From these judgments the soul formed ideas of these things, transporting the sensations of heat, color, and so on, to the things themselves outside the soul. These are *the obscure and confused* ideas we have of sensible qualities, the soul adding *its false judgments* to what nature caused it to know.[37]

As children we form the habit of believing falsely that *S is P* is true. This habit in turn leads us to form the complex false idea *SP* by combining the comprehensions of *S* and *P*. It follows that any proposition of the form *SP is Q* is false because its subject term fails of existential import. An example discussed is *corporeal pain*. As a children we falsely believe *fire causes pain*. This habit leads us to form the false idea *corporeal pain*. The proposition *corporeal pain is in my head* is then false because the subject *corporeal pain* fails to signify anything in the world. Other examples discussed include *heat caused by fire*, *gravity*, and *happiness caused by material wealth*.[38]

An important consequence of the theory false idea is that the *Logic* understands affirmative propositions to carry existential import. That they do so was a standard teaching of the logic of the day.[39] The assumption is easily incorporated into the referential reading of extension: *every S is P* is true if, and only if, *S* signifies at least one thing and the extension of *S* is included in that of *P*. Because the *Logic* subscribes, in addition, to the standard view that universal affirmatives entail particular affirmatives, the subject term of particulars also carries existential import.[40] Indeed, it may be doubted that the authors of the *Logic* thought much about the properties of the empty set, but

[37] *LAP* I,9, *KM* V,157–158, *B* 49–50.

[38] For passages in which the formation of such ideas are described see: *LAP Discour* I, *KM* V, 110, *B* 9–10; *LAP* I,9. *KM* V, 157–78; *B* 49–50; I,11. *KM* V, 168–170; *B* 58-60.

[39] Ashworth, E. J. 1973 (1973). "Existential assumptions in late medieval logic", *American Philosophical Quarterly* 10, pp. 141–147.

[40] Based ln his intentional reading of extension Jean-Claude Pariente (Pariente, Jean-Claude (1985). *L'analyse du language à Port-Royal*, Paris, C.N.R.S. Éditions de Minuit. pp. 246–

from the perspective of modern theory, there would be an obvious reason for incorporating existential import into the truth-conditions of affirmatives under the referential reading of extension, for if otherwise, any propositions with a false idea as subject would be true because its extension would be empty, and therefore a subset of the extension of any predicate.

The intentional reading of extension, on the other hand, seems to be committed to the truth of many universal affirmatives with false ideas as subjects. On that reading, the comprehension of the restriction of S by P is the union of the comprehension of S and P. Moreover, *every S is P* is true if the comprehension of S is a subset of that of P. It follows then that *a Chimera is a Chimera* and *every golden mountain is golden* is true. On the referential reading, however, such propositions would be false because their subject terms fail of signification. It is true that there was a long tradition in medieval logic that held that such propositions displaying "essential truths" are true without carrying existential import, either because they are disguised logical entailments (*consequentiæ*) or because their subject matter is not things in the world but some sort of intentional entity or "objective being." Despite holding a sim-

247) argues that neither universal nor particular affirmatives carries existential import. He, therefore, rejects the reading of the in which propositions with false ideas as subjects are false because their subject fails to signify. His argument for his interpretation is based on what he cites as a counter-example to the claim that affirmatives with false ideas as subjects are always false. This is the *Logic*'s analysis at II:7 of the proposition *Alexander, who was the son of Philip, is defeated the Persians*. The case under discussion in that in which the *Logic* posits that *Alexander, who was the son of Philip, is defeated the Persians* is true, *Alexander was the son of Philip* is false, *Alexander defeated the Persians* is true, and that the subject term *Alexander, who was the son of Philip* is a case of "explication." Pariente argues that this is the desired counter-example because the subject term *Alexander, who was the son of Philip* is a false idea but the proposition as a whole *Alexander, who was the son of Philip, defeated the Persians* is true. Pariente, however, misreads the example. There are two sorts of restriction, *determination* and *explication* (I:8) Determination is that in which the comprehension of the restricted complex term is the intersection of the comprehensions of that of the head noun and the relative clause. Today we call such a relative clause *restrictive*. Explication is that in which the comprehension of the complex restricted term is that of the head noun; the modifying clause does not alter the comprehension of the head noun. Today we call such a clause *non-restrictive*. The whole point of the *Logic*'s discussion is that because the restriction is explication rather than determination the subject is in fact not a false idea. It has the same comprehension as Alexander. According the whole proposition *Alexander, who was the son of Philip, defeated the Persians* has the same truth-value as the part *Alexander defeated the Persians*. The same point can be made by saying that in mental language two propositions are being asserted, *Alexander was the son of Philip* and *Alexander defeated the Persians*. *Ex hypothesis*, the former is true and the latter is false, but the subject term in both signifies Alexander. In neither is it a false idea. False ideas are irrelevant. It should be remarked that the analysis ("exposition") of propositions with non-restrictive subject terms as a conjunctive assertion, which in no way appeals to the notion of false idea, was not new to the *Logic*. See, for example, Buridan, John (2001). *Summulæ de Dialectica*, New Haven, Yale University Press. 4.4.5.

ilar view to that of the *Logic* that propositions with false ideas are subjects are false, Descartes also holds that there are eternal essential truths with false ideas as subjects.[41] The *Logic*'s authors, however, never address cases in which a universal affirmative states an essential definition of a false idea, like *a chiliagon has a thousand sides*. Given their vagueness it is possible that they held two parallel theories of truth, one for essential truths and another contingent truths.[42]

9 Conclusions

It is true that the intentional interpretation of extension provides a direct and plausible account of the truth-conditions of essential truths, a class that includes real definitions, necessary truths, and what the *Logic* considers to be scientific truths. Like The referential reading, however, also explains essential truths. Any proposition that is true on the intentional reading is also true on the referential reading. On the other hand, the referential reading explains a significant list of features that the intentional reading cannot. First, it explains what contingent truths are and the conditions under which they are true, an issue the intentional reading does not address. Second, the referential reading forms the background of the *Logic*'s account of how sensation justifies knowledge of contingent truths. Third, it provides the right truth-conditions for categorical propositions. Lastly, it easily incorporates into its truth-conditions the requirement of existential import necessary to the *Logic*'s account of false ideas and error. One issue on which the interpretations diverge and on which the text of the *Logic* offers no help is the truth-value of essential truths with subject terms that are false ideas. The intentional interpretation entails that they are true, and the referential that they are false. I suspect that this is an issue with which the authors would have little patience. The questions of whether *a chimera is a chimera* is now true and whether *every man is an animal* was true before creation is exactly the sort of scholastic philosophy they intentionally avoided.[43]

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[41] *Meditation* V.05, *AT* 7.64, 76–77. English translations of the *Meditations* are from Descartes, René (2007–2010). *Meditations on first philosophy*, MS.

[42] For an extensive discussion of the issue see Martin, John N. (2011). "Existential import in cartesian semantics", *History and Philosophy of Logic* 32:2, pp. 1–29.

[43] See *Discours* I, *KM5*:112-113, *B* 11-12; IV:6 *KM5*:380, *B* 249.

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