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Awareness and Creativity by Evolutive EPM

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Abstract

In the last fifty years, many experiments made by psychologists of reasoning have often shown most adults commit logical fallacies in propositional inferences. As a matter of fact, an important component of human rationality resides in the diagram of the squares of opposition, as formal articulations of logical dependence between connectives. But the formal rationality provided by the squares is not spontaneous and therefore, should not be easy to learn for adults. This is the main reason why we need reliable and effective training tools to achieve full logic proficiency and predicative competence, like the Elementary Pragmatic Model (EPM). EPM extension as "Evolutive Elementary Pragmatic Model" (E^2PM) represents a contribution to current modelling and simulation, offering an example of new forms of evolutive behavior by inter- and transdisciplinarity modelling (e.g. strategic foresight, uncertainty management, embracing the unknown, creativity, etc.) for the children of the Anthropocene.

Keywords: elementary pragmatic model, precise reasoning, universal logic, Klein four-group, computational information conservation theory, ontological uncertainty management, cognitive neuroscience, cognitive informatics, brain–inspired systems, cybersemiotics, EEPM, E^2PM, CICT.

Introduction

In every discourse, whether of the mind conversing with its own thoughts, or of the individual in his intercourse with others, there is an assumed or expressed limit within which the subjects of its operation are confined. The most unfettered discourse is that in which the words we use are understood in the widest possible application, and for them the limits of discourse are co-extensive with those of the universe itself. But more usually we confine ourselves to a less spacious field. Sometimes, in discoursing of human beings we imply (without expressing the limitation) that it is of human beings only under certain circumstances and conditions that we speak, as of civilized men, or of human beings in the vigor of life, or of human beings under some other condition or relation. Now, whatever may be the extent of the field within which all the objects of our discourse are found, that field may properly be termed the "universe of discourse" ([8], p.42.) This concept, probably created by Irish mathematician, educator, philosopher and logician George Boole (b.1815d.1864) in 1847, played a crucial role in his philosophy of logic especially in his stunning principle of "wholistic reference" [12, 13].

The term "universe of discourse" generally refers to the collection of symbolic objects being managed and discussed in a specific discourse. In current model-theoretical semantics, a universe of discourse is the set of symbolic entities that a model is based on. Furthermore, this universe of discourse is in the strictest sense the ultimate subject of the discourse and human ability to use logic, to integrate the evidence of our senses in a noncontradictory way, is part of our rational faculty, the very faculty that makes us human. Attempts to introduce semantics into information theory to arrive at cybersemiotics have made some progress but fell short of having a capability to deal with information described in natural language. Obviously, we also have the capacity to be illogical, but that is because our rational faculty also entails volition, the power to choose to think or not to think.

A fundamental concept plays a key role in human intelligence. A concept whose basic importance has long been and continues to be unrecognized. The concept of "restriction" is pervasive in human cognition. Restrictions underlie the remarkable human ability to reason and make rational decisions in an environment of imprecision, uncertainty and incompleteness of information. Such environments are the norm in the real-world. Such environments have the traditional logical systems that become "dysfunctional". The concept of "restriction" is really close to the concept of "closure space". The concept of "closure space" was developed around 1930 by Polish logician, mathematician Alfred Tarski (b.1901d.1983), who conceived an abstract theory of logical deductions which models some properties of logical calculi. Therefore, the concept of "restriction" can open the door to an effective theory of cybersemantic information processing [98]. There are many applications in which cybersemantics of information plays an important role. Among such applications are: machine translation, summarization, search, decision-making under uncertainty, cognitive informatics, etc.

According to Swiss clinical psychologist Jean Piaget (b.1896d.1980), human adults normally know how to use properly classical propositional logic. Piaget also held that the integration of algebraic composition and relational ordering in formal logic is realized via the mathematical Klein group structure

[53]. In the last fifty years, many experiments made by psychologists of reasoning have often shown most adults commit logical fallacies in propositional inferences [55]. These experimental psychologists have so concluded, relying on many empirical evidences, that Piaget's claim about adults' competence in propositional logic was wrong and much too rationalist. But, doing so, they forgot Piaget's rigorous and important analysis of the Klein group structure at work in logical competence. In other words, according to experimental psychologists, Piaget was overestimating the logical capacities of average human adults in the use of classical propositional logical connectives. As a matter of fact, English speaking people tend to treat conditionals as equivalences and inclusive disjunctions as being exclusive [77].

Nevertheless, the Klein group structure Piaget used can be reused to help us understand better what happens in spontaneous human reasoning and in the production of fallacies [55]. In fact, in mathematics, the Klein four-group or "Vierergruppe", named by German mathematician Felix Klein (b.1849 d.1925) in 1884, is a group of four transformations with four elements. The Klein four-group is the smallest non-cyclic group, and every non-cyclic group of order 4 is isomorphic to the Klein four-group. The cyclic group of order 4 and the Klein four-group are therefore, up to isomorphism, the only groups of order 4. Both are abelian groups in mathematics. Piaget applied the Klein four-group to binary connectives, so that a given connective is associated first with itself (in an identical (I) transformation) and then with its algebraic complement (its inverse (N) transformation or additive complement), also with its order opposite (its reciprocal (R) transformation or multiplicative complement) and finally, with the combination of N and R transformations (that Piaget calls "correlative" or C transformation) ([53], ch.17.) This correlative corresponds to what logicians usually call the "dual" (D) transformation [77].

The Klein four-group structure generates squares of opposition (SOO), and an important component of human rationality resides in the diagram of the SOO, as formal articulations of logical dependence between connectives. The origin of the SOO can be traced back to Aristotle making the distinction between two oppositions: contradiction and contrariety. But Aristotle did not draw any diagram. This was done several centuries later by Apuleius and Boethius in the second and sixth centuries. So, SOOs are considered as important basic components of logical competence and of human rationality [6]. Treating conveniently neutral element (I), algebraic complement (N) and order reciprocal (R) in an integrated structure, by a valid treatment of dual (D), would guarantee people to make logically valid classical inferences on propositions. But the formal rationality provided by the SOOs is not spontaneous and therefore, should not be easy to learn for adults. This is the main reason why we need reliable and effective training tools like the Elementary Pragmatic

Model (EPM) to achieve full logic and predicative proficiency.

1 The Elementary Pragmatic Model (EPM)

Automatic vs. controlled, convergent vs. divergent, implicit vs. explicit, reflexive vs. reflective, etc. processing correspond to theoretical cognitive dichotomies that have been around for a few generations and have contributed to the development of many neurocognitive models and systems in the past century. Among them, the relational "Model of the Rational Mind" allows the adoption of a different perspective from that of traditional psychology. It was named the "Elementary Pragmatic Model" (EPM) by developers, Italian mathematician Alberto Silvestri (b.1942-d.1986) and Italian psychologist and psychiatrist Piero De Giacomo (b.1935-) in the late 1960s [18, 19, 63, 85]. At first it was conceived to explore interpersonal relationships. *EPM* can be conceptualized through an adapted case study published in "Finite Systems and Infinite Interactions" [16]. *EPM* was developed following Gregory Bateson's constructivist participant observer concept in the "second order cybernetics", to arrive to what was called "new cybernetics", according to the von Foerster's classical, historical categorization of cybernetics in 1974 [43, 83]. In Italy, *EPM* was first described in full in 1979 [18], and a complete description of clinical applications was then made in the course of the 1980s, 1990s and 2000s [16, 23, 62].

Thanks to its natural language interface, initially *EPM* has been used as a theoretical family therapy model to classify the outcomes of dyadic interactions in psychology. It was used successfully by a group of therapists in family therapy and in clinical psychiatric training and applications (e.g. schizophrenia, nervous anorexia, etc.) Later it was applied to develop interactive psychotherapeutic strategies, online counseling, E-therapy and E-learning. Access to online education via Internet, even by smartphones, provides educational leaders of tomorrow opportunities for personal and professional growth. Education via wireless networks will undoubtedly open up new doors for the development of interdisciplinary studies such as integrated arts and writing and even online writing interventions immersed in avatar experiences. Since the beginning of the new millennium, application areas for *EPM* have been extended to other disciplines and even to engineering applications like user modelling, constraint requirements elicitation, software creativity and adaptive system design and development [21, 24, 60, 69, 80, 81]. Given the state of the development for Natural Language Processing (NLP) systems, heavyweight tools in isolation are not effective in supporting conceptual model construction. There may still be a role for them as an adjunct to the techniques the author will describe. Instead, it makes sense to adopt lightweight linguistic tools that can be tailored to particular linguistic analysis tasks and scale up. Moreover, linguistic analysis may be more useful for large textual documents that need to be analysed quickly (but not necessarily very accurately), in contrast to shorter documents that need to be analysed carefully [60].

It is important to underline that information processing technology can be used also to facilitate the application of a pragmatic model to "prescribe" or suggest to participants to improve their attitudes, their creativity. De Giacomo and Silvestri defined a model that introduces the elementary pragmatic as the unit of description of participants' interactions. An elementary pragmatic interaction involves two participants that exchange bits of "pragmatic" information in terms of acceptance or refusal of a topic or proposal and can be formalized as triples of Boolean values logical structure quite close to Matte Blanco's triad [67], according to which it is the entity formed by two theoretical objects related each other by a third object called "relation". *EPM* has shown to be a highly operative and versatile tool and new application areas are continually envisaged [21, 22]. Recently, the *EPM* intrinsic Self-Reflexive Functional Logical Closure has even contributed to finding an original solution to the dreadful "Information Double-Bind" (IDB) problem in classic science [28, 35].

Previously, we already noticed that the concept of "closure space" was developed around 1930 by Tarski, who conceived an abstract theory of logical deductions which models some properties of logical calculi. Tarski's undefinability theorem shows that Gdel's arithmetization encoding cannot be done for semantical concepts such as "truth". It shows that no sufficiently rich interpreted language can represent its own semantics. Mathematically, what he described is just a finitary closure operator on a set (the set of sentences). In Logic, the structure of closure spaces is defined by the "consequence operator" introduced by Tarski.

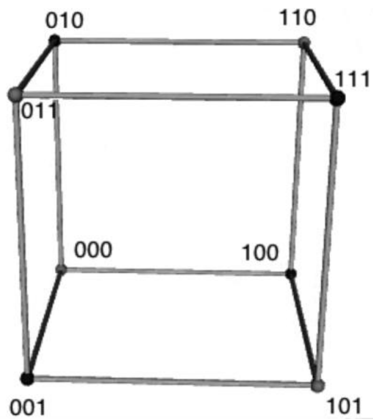

Figure 1: EPM associated Boolean algebra B_3 is represented by cube C_3 with its bitstring decoration (LTR) in $\mathbb{R}^3$.

By the mathematical point of view, the classic EPM associated Boolean algebra B_3 can be represented Left-To-Right (LTR) by a cube C_3 in three-dimensional Euclidean space $\mathbb{R}^3$ (Figure 1). This is done by the "conventional" LTR (Left-To-Right) coordinate mapping $c : \{0; 1\}^3 \to \mathbb{R}^3$. Any Boolean algebra of order 2^n, called B_n, is graded of rank n [61], and can be represented as a hypercube (HC) or n-cube C_n, in n-dimensional Euclidean space $\mathbb{R}^n$, for $n = 0, 1, 2, 3, 4, \cdots, \infty, n \in \mathbb{N}$. Then, classic EPM can be extended to E^2PM) or $EEPM$ (Extended or Evolutive Elementary Pragmatic Model) by providing traditional EPM with a graded–controlled, incremental open–closed logic architecture through the asymptotic process of a structured sequence of locally finite Boolean algebras, theoretically for $n \longrightarrow \infty$ [17].

The process to obtain successive n-dimensional HCs can be formalized mathematically as a Minkowski sum: the n-dimensional HC is the Minkowski sum of n mutually perpendicular unit-length line segments, and is therefore an example of zonotope.

The n-dimensional HC geometrical information can be projected to convenient projection planes to study the local behaviour of their connection components as graphs (Petrie Polygon Orthographic Projections, Hasse diagrams, etc.) The graph of the n-HC's edges is isomorphic to the Hasse diagram of the HC $(n-1)$-simplex's face lattice. Such a diagram, with labeled vertices, uniquely determines the set partial ordering. Hasse diagrams were originally devised as a technique for making drawings of partially ordered sets (posets) by hand. They have more recently been created automatically using graph drawing techniques. Although Hasse diagrams are simple as well as intuitive tools for dealing with finite posets, it turns out to be rather difficult to draw "good" diagrams. The reason is that there will in general be many possible ways to draw a Hasse diagram for a given poset, according to different viewpoints. The simple technique of just starting with the minimal elements of partial order and then drawing greater elements incrementally often produces quite poor results: symmetries and internal structure of the order are easily lost.

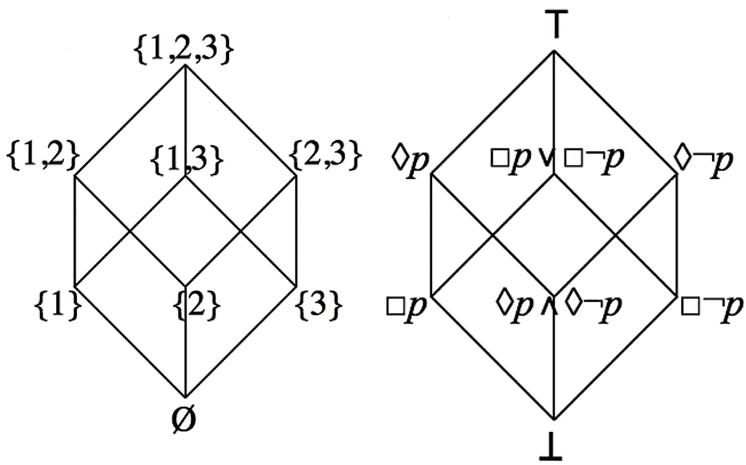

Figure 2: Hasse diagrams for the Boolean algebra $\phi(\{1,2,3\})$ on the left side and for a Boolean algebra of formulas from the modal logic $S5$ with modality $(\Box, \Diamond)$, on the right side [26].

Nevertheless, it is well-known that a Boolean algebra can always be visualized by means of a Hasse diagram that is centrally symmetric, with all complementary pairs of elements ordered around the center of symmetry [15]. Furthermore, a finite Boolean algebra can always be partitioned into "levels" $L_0, L_1, L_2,,$ which are recursively defined as follows: $L_0 = [\perp]$, and

$$L_{k+1} = \{x \mid \exists y \in L_k : y \lhd x\}. \tag{1}$$

Hasse diagrams for the Boolean algebra $\phi(\{1,2,3\})$, and for a Boolean algebra of formulas from the modal logic $S5$ are shown in Figure 2. It is immediate to map the cube C_3 to Hasse diagram on the left side of Figure 2. Boolean algebras are locally finite and their word problem is always decidable (closed logic or full logic closure).

2 Concerted Restriction and Graded Closure

Starting from the concept of "restriction" [98], we can open the door to an effective theory of cybersemantic information definition. In fact we can define "concerted restriction" as "group controlled restriction", to overcome the limitations of traditional definitions by "logical consequence", without the need of any metamathematical trick as in the past approaches. In fact, Tarski's traditional conceptual analysis of the notion of "logical consequence" is one of the pinnacles of the process of defining the metamathematical foundations of mathematics in the tradition of his predecessors Euclid, Frege, Russell and Hilbert, and his contemporaries Carnap, Gdel, Gentzen and Turing. In metamathematics Frege's "Begriffschifft" provided the material for a revolution, but Hilbert's "Grundlagen" took it far beyond what he intended, and Tarski's axiomatization of logical consequence consolidated the new paradigm. His formulation of the nature and definition of the logical supports Hilbert's stance, and his analysis of "truth" requires only a very weak link between "explicatum" and "explicandum" that leaves open the possibility that further conceptual analysis will result in additional connotations and associated axioms. Tarski also noted that in defining the concept of consequence "efforts were made to adhere to the common usage of the language of every day life" ([94], pp. 409-420.) Tarski's analysis, and Bziau's further generalization of it into universal logic have to reasoning in the everyday lives of ordinary people, from the cognitive processes of children through to those of specialists in the empirical and deductive sciences [5]. What value might a universal logic perspective [4, 5] provide to theoretical and empirical studies of human reasoning? It can be explained by three fundamental components: abstract, conceptual and applied. At an abstract level it focuses on consequence operators as the primary unit of analysis in modelling reasoning, and represents them as relations on the powerset generated by an arbitrary set, with no a priori constraints on what constitutes the elements of the set, algebraic constraints on admissible and equivalent sets, or axioms constraining them. At a conceptual level, a universal logic perspective deconstructs the notion of logical reasoning, directing attention to each assumption made in any logical system, the implications of making it, the interactions between various assumptions, and so on. At an applied level, questions raised within the human reasoning literature across many disciplines might be clarified within a universal logic framework [42]. In universal logic, finitary closure operators are still studied under the name "consequence operator", which was coined by Tarski. Nowadays the term can refer to closure operators which need not be finitary; finitary closure operators are then sometimes called "finite consequence operators". A universal logic approach could model the relationship between the consequence operators resulting from incorporating successively richer connotations of the "explicandum" into the "explicatum". The dynamics of moving the explicatum closer to the explicandum may be modelled within an universal logic framework as one of determining whether additional statements about the explicandum are consequences of the explicatum or, if not, consistent with it, and adjusting the explicatum to remove inconsistency and derive the desired consequences. This is also Piaget's [74, 75] "equilibration" process of human development as "assimilation" and "accommodation" which can be modelled within a logical framework as one of evolutionary theory of change [51].

Certainly one definition of the "explicatum" or "explicandum" may be better than another, but even excellent definition cannot capture all the root properties of the informal domain or application out of which it emerges and interacts within (Figure 3). It structures the informal situation in a more biased way only. So, contrary to any classic scientific tradition, computational information conservation theory (CICT) [28, 35] gives no formal definition in the usual word sense. It does not work by writing formal definitions and deducing inevitable consequences, as many academic researchers tried in the past by the usual, axiomatic top–down approach. In fact, in the past decades, from that approach we learned only how traditionally defined human–made system can be quite fragile to unexpected perturbation, because statistics by itself can fool you unfortunately. Therefore, rather than pretending any definitive, rigorous word definition, CICT simply applies a bottom–up approach, by using technoscience from below by a transdisciplinary, deep learning approach.

In fact, we need to be deeply aware of the fundamental operative difference between approximated and exact precision closure. In this way, we arrive to the concept of "graded closure" representation by arbitrary exact precision, according to CICT. Using this conceptual difference, we can develop an effective theory of cybersemantic information definition by optimized numeric words as patterns linked to "thought patterns of exactingly perfect order", remembering Star Trek V'ger–Voyager logic capabilities definition by Spock [91].

As an example, recently, the IDB problem has been addressed by CICT. As a consequence, one of the first practical result has been to realize that the classical experimental observation process, even in highly ideal operative controlled condition, like the one achieved in current, highly sophisticated and advanced experimental laboratories like CERN in Geneva [10], can capture just a small fraction of the total ideally available information from a unique experiment. Usually, the remaining, major part is completely lost and inevitably dispersed through environment into something we call "background noise" or "random noise" usually, in every scientific experimental endeavor [28]. The same understanding can be used even to model the "coherence–decoherence" transition from quantum to classical system [31].

In turn, this new awareness has forced the scientific community to develop new tools to quantify the information dissipation process in advanced instrumentation [29, 30] and has allowed researchers to enlarge their panorama for neurocognitive system behavior understanding to develop information conservation and regeneration systems in a numeric self-reflexive/reflective evolutive reference framework [28]. Accordingly, new methods and models for building effective applications and strategies can be conveniently conceived, based on new forms of inter- and transdisciplinary understanding [32, 33, 34, 35].

Thus, current biomedical and complex system cybernetics operative knowledge can reach "fourth order cybernetics" level, where multiple realities can emerge by the freedom of choice of the creative observer determining in large part the outcome for both the system and the observer [20, 36]. This places deep demands on the self-awareness of the observer, and on his/her "response-ability" for/in action. At the quantum level, there is entanglement of photons, electrons, particles, atoms, etc. In the classical wave/particle experiments, the observer determines the outcome of the experiment by his choices. Thus, there is the argument that consciousness/observer/intent dictates the outcome of what is manifested in the wave/particle experiments. To model that kind of behavior, a logically closed model cannot cope with ontological uncertainty effectively, so an EPM extension is needed to offer a convenient complementary logical aperture operational support. In fact, EPM can provide us with a reliable self-reflexive closed logic starting scheme to face "unknown known" situations first [17, 20, 37]. Then, if we wish to use it on more and more complex applications with ability to capture even natural emergent phenomena dynamics, EPM must be extended with a natural open logic architecture to face unpredictable interaction and perturbation ("unknown unknowns") at the system design level (Figure 3).

Following this approach, an evolutive framework to manage unexpected dynamics can be developed for EPM. To reach this goal requires starting from traditional mankind worldview to arrive at a convenient ontological uncertainty management (OUM) solution, from which the final EPM extension can be designed [20]. As a matter of fact, the extension is achieved by applying to EPM the CICT anticipatory learning system (ALS) concept [36, 37]. In this way, it is possible to develop an EPM extended framework able to profit from both classic EPM intrinsic "Self-Reflexive Functional Logical Closure" and new, coupled CICT numeric "Self-Reflective Functional Logical Aperture" to obtain the overall extended or evolutive EPM (EEPM or E^2PM) system model. In logic, diagrammatic notation is the main form of knowledge representation, bypassing a deductive procedure through the process of spatial inference. In this evolutive framework, classic EPM can be thought as a reliable starting subsystem to face "unknown knowns" situations and to initialize a process of continuous self-organizing and self-logic learning system refinement to learn "unknown unknowns" (lower-right square in Figure 3). Before proceeding to EPM extension main logic, geometric and numeric properties, a brief recap on duality in logic geometry and language is needed to better understand the presented final results in Section 4.

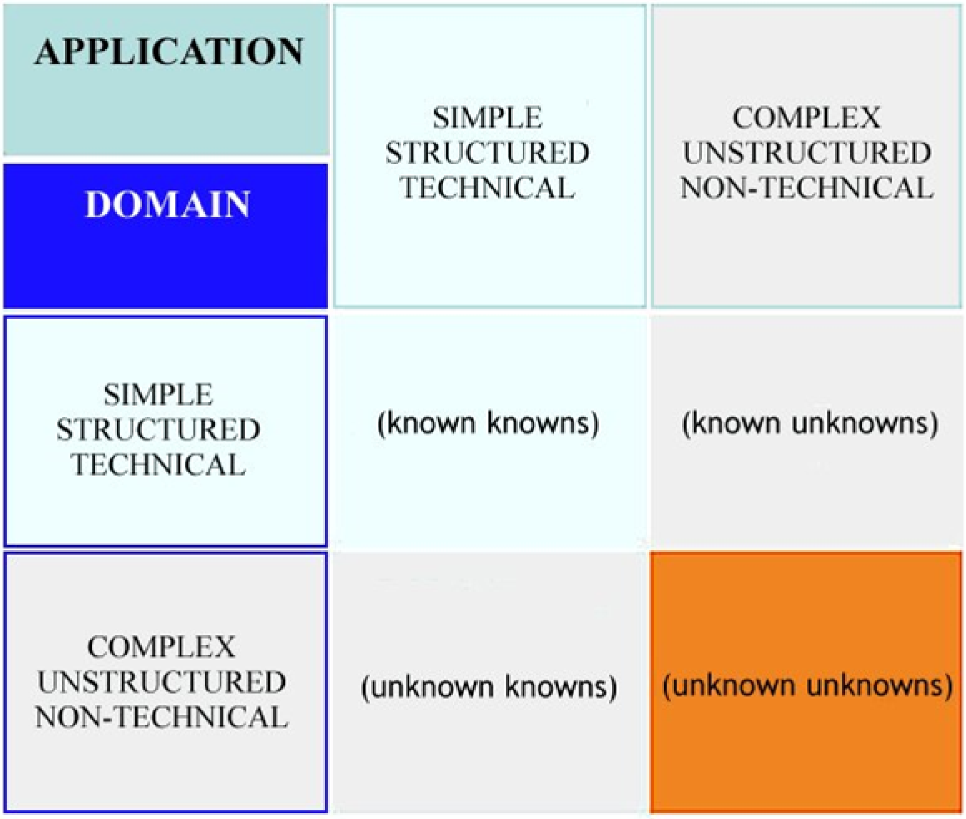

Figure 3: CICT Four-Quadrant Scheme (FQS) for Application and Domain Unknowns.

3 Duality in Logic Geometry and Language

For understanding E^2PM fundamental properties, it is convenient to refer to Order Theory (OT). OT is a branch of mathematics which investigates the intuitive notion of order using binary relations [7]. In OT a Hasse diagram is a type of mathematical diagram used to represent a finite partially ordered set, in the form of a drawing of its transitive reduction where its power set is a graded poset. These are graph drawings where the vertices are the elements of the poset and the ordering relation is indicated by both the edges and the relative positioning of the vertices. Orders are drawn bottom-up: if an element x is smaller than (precedes) y then there exists an upward-directed path from x to y. It is often necessary for the edges connecting elements to cross each other, but elements must never be located upon an edge (e.g. Figure 2).

A concept can be defined by just inverting the ordering in a prior definition. This is the case for "least" and "greatest", for "minimal" and "maximal", for "upper bound" and "lower bound", and so on. This is a general situation in OT: a given order can be inverted by just exchanging its direction, pictorially flipping the Hasse diagram top-down (the representation is reversible). This yields the so-called dual inverse, or opposite order. Every OT definition has its dual: it is the notion one obtains by applying the definition to the inverse order. Since all concepts are symmetric, this operation preserves the theorems of partial orders. If a given statement is valid for all partially ordered sets, then its dual statement, obtained by inverting the direction of all order relations and by dualizing all order theoretic definitions involved, is also valid for all partially ordered sets.

If a statement or definition is equivalent to its dual then it is said to be "self-dual". A simple example in natural language is a palindrome. A palindrome is a word, phrase, number, or other sequence of characters which reads the same backward or forward. Allowances may be made for adjustments to capital letters, punctuation, and word dividers. Examples in English include "A man, a plan, a canal, Panama", "Amor, Roma", "Was it a car or a cat I saw?", etc. A simple example in numeric language is given by numeric words obtained by powers of eleven represented in decimal base (root system). They are numeric palindromes. Note that the consideration of dual orders is so fundamental that it often occurs implicitly when writing "$\geq$" for the dual order of "$\leq$" without giving any prior definition of this "new" symbol.

Duality phenomena occur in nearly all mathematically formalized disciplines, such as algebra, geometry, logic and natural language semantics. However, many of these disciplines use the term "duality" in vastly different senses, and while some of these senses are intimately connected to each other, others seem to be entirely unrelated. Consequently, if the term "duality" is used in two different senses in one and the same work, the authors often explicitly warn about the potential confusion.

Specifically, in mathematics, there is an ample supply of categorical dualities between certain categories of topological spaces and categories of partially ordered sets. Today, these dualities are usually collected under the label "Stone duality", since they form a natural generalization of Stone's representation theorem for Boolean algebras. The theorem was first proved by American mathematician Marshall H. Stone (b.1903d.1989), and thus named in his honor. Stone was led to it by his study of the spectral theory of operators on a Hilbert space [93].

In turn, the well-known Stone duality between Stone spaces and Boolean algebras is generalized by the "duality theory for distributive lattices". It provides three different (but closely related) representations of bounded distributive lattices via "Priestley spaces", "spectral spaces", and "pairwise Stone spaces" [76]. A Priestley space is an ordered topological space with special properties. In particular, there is a duality between the category of Priestley spaces and the category of "bounded distributive lattices". Then, "abstract algebraic logic" provides the appropriate theoretical framework for developing a uniform duality and canonical extensions theory for non-classical logics. Such theory serves, in essence, to define uniformly referential (e.g. relational, Kripke-style) semantics of a wide range of logics for which an algebraic semantics is already known [38].

Stone-type dualities also provide the foundation for "pointless topology" (PT) and are exploited in theoretical computer science for the classic study of formal semantics. PT (also called point-free or pointfree topology) is an approach to topology that avoids mentioning points. "Locales" and "frames" form the foundation of PT, which, instead of building on point-set topology, recasts the ideas of general topology in categorical terms, as statements on frames and locales [96]. The name "pointless topology" is due to Hungarian-American polymathematician John von Neumann (b.1903-d.1957). The ideas of PT are closely related to "mereotopologies" in which regions (sets) are treated as foundational without explicit reference to underlying point sets. In "formal ontology", a branch of metaphysics, and in "ontological computer science", mereotopology is a first-order theory, embodying mereological and topological concepts, of the relations among wholes, parts, parts of parts, and the boundaries between parts [95].

In 1969 George Spencer-Brown published his ground-breaking book "Laws of Form" and suggested that in logic a temporal dimension was needed [90]. He suggested that there should be imaginary values in logic analogous to the square root of minus one in mathematics. He pointed out that the paradoxical logical value x such that $x =\sim x$ (x is its own negation; x is true if and only if x is false and so can be neither true nor false) could be interpreted as such an imaginary. He further pointed out that the solution to the paradox inherent in $x =\sim x$ lies in the temporal dimension [95]. Since then, everything has evolved in Logic, opening many working themes such as how Self-reference and Recursive Forms, Biology and Logic, Time and Paradox, etc. are mutually related [57, 58, 59].

CICT's new awareness of a discrete hyperbolic geometry (HG) subspace (reciprocal space, RS) of coded heterogeneous hyperbolic structures [28], underlying the coupled, familiar rational $\mathbb{Q}$ Euclidean (direct space, DS) surface representation allows to achieve full information conservation. This is the main reason why CICT is the natural framework for arbitrary multiscale computational modelling in the current landscape of modern computational Quantum Field Theory (QFT) [32, 33, 34, 35]. The fundamental CICT assumption is that the play of human information observation interaction with an "external world representation" is related by the different manifestation and representation properties of a unique fundamental computational information structuring principle: the Kelvin Transform (KT) [66]. KT is key to efficient scale related information representation, structuring "external space" information to a dual "internal representation" and vice-versa by projective-inversive geometry [29]. As a matter of fact, Euclidean geometry is a subset of a whole family of non-Euclidean geometries through a range of curvatures both positive and negative. The relationship between a point in three-dimensional space and its conformal reflection within the conformal sphere is exactly like the relation between a point on the number line outside of unity and its reciprocal within the bounded unitary range. The reciprocal function maps the three-dimensional space beyond unit distance out to infinity in all directions to its conformally mapped spatial reciprocal located within the finite bounded volume of the unit sphere. "Animals and humans use their finite brains to comprehend and adapt to an infinitely complex environment" [39]. According to CICT, KT is the fundamental tool to build concerted restriction with graded closure rational map representations of the Real we are immersed within and we are part of [29, 32].

4 E^2PM Fundamental Logic Geometry Properties

The HC is just one of three families of regular polytopes that are represented in any number of dimensions. In n-space (for $n > 4$), there are exactly three regular n-dimensional polytopes, the n-simplex, the n-cube, HC, and the geometric n-dimensional cube-dual, the n-octahedron (HyperOctahedron, HO). There are no further regular polytopes [14]. Considering the infinite family of n-cube C_n, in n-dimensional Euclidean space $\mathbb{R}^n$, for $n = 0, 1, 2, 3, 4, \cdots, \infty, n \in \mathbb{N}$, their related power set $P(B_n)$ Hasse diagrams graded posets can be grouped into two large families, according to the oddness or eveness of their dimension index n.

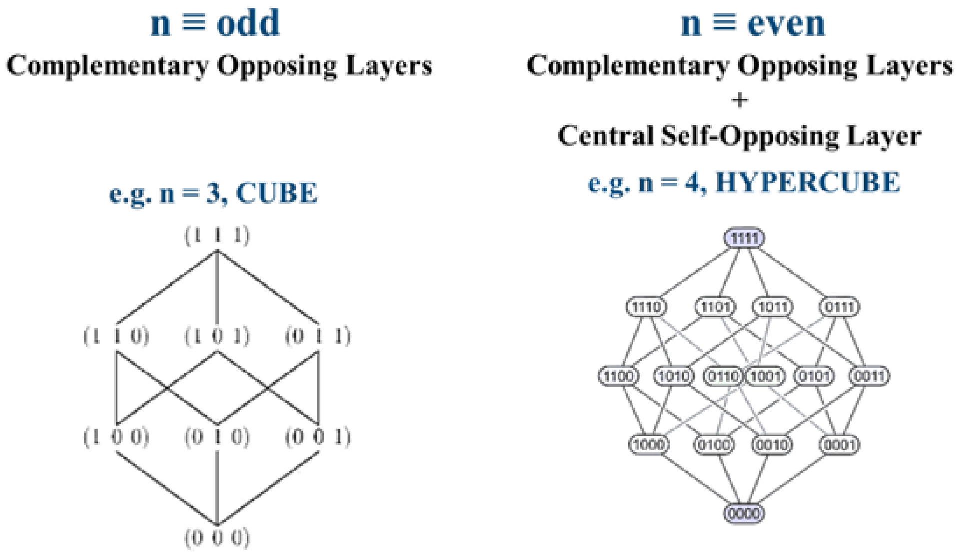

Figure 4: Power Set $P(B_n)$ Hasse Diagrams n-cube C_n Graded Posets Can be Grouped into Two Large Families (see text).

As an example, in Fig.4, HCs are depicted for $n = 3$ (odd) and $n = 4$ (even). For even index, a central self-complementary layer (in this case formed by $A(4) = 6$ terms) divides the Hasse diagram into two complementary half-diagrams. For odd index, Hasse diagram can be thought as the union of two complementary half-diagrams, with no self-complementary layer.

The power set $P(B_n)$ of any locally finite Boolean algebra B_n (n even) can be thought as a Self-Complementing (Reflective) Functional Logical Closure for the power set $P(B_{n-1})$ of preceding locally finite Boolean algebra B_{n-1}. According to CICT, this property is fundamental to achieve overall model systemic resilience and antifragility behavior [28, 30, 35].

In general, the number of terms $A(n)$ composing the central self-complementing layer is given by the following Central Binomial Coefficients $C(2n, n)$ Series $A(n)$:

$$A(n) = \left(\frac{(n-2)!2\,(n-1)}{\left(\frac{(n-2)}{2}\right)!\left(\frac{(n)}{2}\right)!} \right) for \;\; n = 2, 4, 6, \ldots (n \;\; even). \qquad (2)$$

In Figure 5 further computational detail are available to the interested reader. Furthermore, it is possible to compute the Central Self-Opposing Layer Central Binomial Coefficients $C(2n, n)$ Series $A(n)$ Asymptotic Convergence

Speed Limit SL_∞ , given by:

$$SL_\infty = \lim_{n\to\infty}\left(\frac{A(n+2)}{A(n)}\right) = \lim_{n\to\infty}\left(\frac{4(n-1)}{n}\right) = \bar{0}4.\bar{0} \ , \tag{3}$$

where the bar over digit means its infinite repetition, as usual for real number $\mathbb{R}$ representation conformity. Convolving $A(n)$ with itself yields the powers of 4 series $P(n)$: 1, 4, 16, 64, 256, 1024, 4096, 16384, 65536, 262144, 1048576, , where $P(n)$ counts the number of compositions of Natural Numbers, $\mathbb{N}$, into n parts, with each part less than 4. The cube and the octahedron form a dual pair. The power set $P(B_n)$ of any locally finite Boolean algebra B_n form a dual pair with any locally finite n-HyperOctahedron algebra n-HO.

Central Self-Opposing Layer Central Binomial Coefficients C(2n,n) Series										
n	2	4	6	8	10	12	14	16	18	...
A(n)	2	6	20	70	252	924	3432	12870	48620	...
										...
m=(n-2)/2	0	1	2	3	4	5	6	7	8	...
2(2m+1)	2	6	10	14	18	22	26	30	34	...
C_m	1	1	2	5	14	42	132	429	1430	...
A(n)=2(2m+1) C_m	2	6	20	70	252	924	3432	12870	48620	...

$$A(m) = \frac{2m!\,2(2m+1)}{m!\,(m+1)!} = 2(2m+1)\,C_m \quad for\ \ m=0,1,2,3,\ldots$$

where C_m is the mth Catalan Number, and

$$A(n) = \frac{(n-2)!\,2(n-1)}{\left(\frac{n-2}{2}\right)!\left(\frac{n}{2}\right)!} \quad for\ \ n=2,4,6,8,\ldots$$

where $n=2m+2$.

Figure 5: Computational details for the Central Binomial Coefficients $C(2n,n)$ Series $A(n)$.

In addition to using diagrams for the visual representation of individual formulae or propositions, logicians also use diagrams to visualize certain relations between formulae from some given logical system. For example, the relations of contradiction, contrariety, subcontrariety and subalternation which hold between a set of logical formulae, are standardly visualised by means of Aristotelian diagrams, such as the well-known SOOs [84]. An Aristotelian Geometry (AG) diagram visualizes a set of logical formulas and the Aristotelian relations between them. The basic logical formulas synthetize four relations for:

"contradiction" (CD), "contrariety" (C), "subcontrariety" (SC), "subalternation" (SA). According to Demey and Smessaert [26] there is a fundamental relationship between Aristotelian logic and Hasse diagrams. Both types of diagrams can be seen as vertex-first projections of n-dimensional HCs as showed in Figure 6, and whether the diagram is Aristotelian or Hasse depends on the choice of the projection axis only [26]. In Hasse diagrams, the implications all go in the same general direction (viz. upwards), but in Aristotelian diagrams, they tend to go in a wide variety of directions.

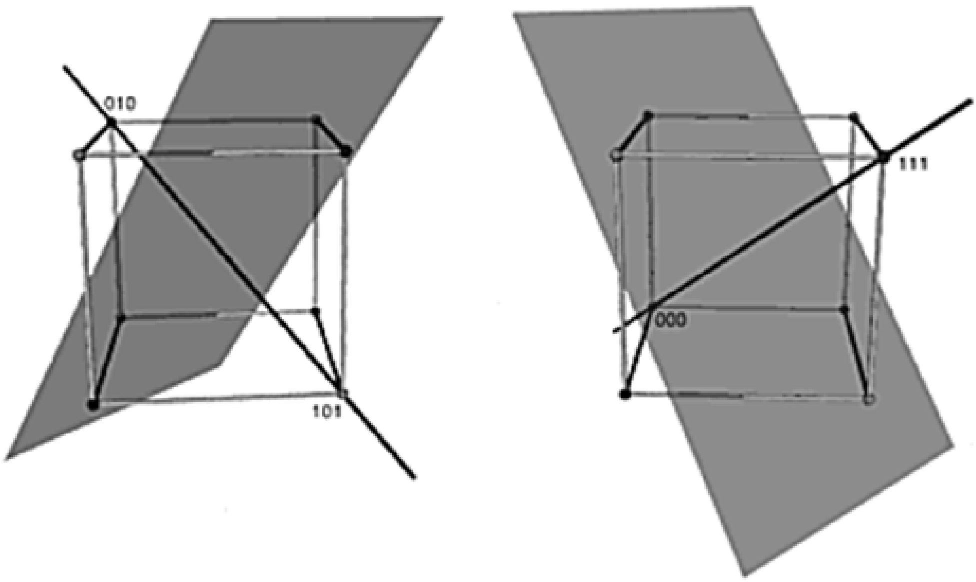

Figure 6: On the left, cube projection axis and projection plane related to Hasse Diagram; on the right, cube projection axis and projection plane corresponding to Aristotelian Diagram [87].

The most widely known Aristotelian diagram is of course the so-called "square of oppositions" (SOO). This diagram has a rich tradition [84], but it is also widely used by contemporary logicians to visualize interesting fragments of systems such as modal logic [9], (dynamic) epistemic logic [64] and deontic logic [68]. There is also a vast literature on Aristotelian diagrams other than the traditional square. In recent years, several three-dimensional Aristotelian diagrams have been proposed. Many authors employ the notion of duality as a mean to describe the specific details of a particular formal or natural language, without going into any systematic theorizing about this notion itself. Next to such auxiliary uses, however, there also exist more abstract, theoretical accounts that focus on the notion of duality itself.

For example, theoretical perspectives address the group-theoretical aspects of duality, or its interplay with the so-called Aristotelian relations. One such 3-D representation visualises the Aristotelian relations between 14 contingent formulae as vertices on the rhombic dodecahedron, henceforth referred to as RDH [25, 86]. Referring to RDH, Smessaert and Demey provide a more unified account of a whole range of Aristotelian diagrams which have so far mostly been treated independently of one another in the literature [87]. They arrive to define the logical geomety of the rhombic dodecahedron of oppositions as the basic logic geometry, which can be associated to the 4-HC [89].

Informally, two formulae F and G are "contradictory" (CD) when they cannot be true together and cannot be false together. They are "contrary" (C) when they cannot be true together but may be false together and "subcontrary" (SC) when they cannot be false together but may be true together. Finally, notice that "subalternation" (SA) is not defined in terms of the formulae being true together or being false together, but in terms of truth propagation: there is a subalternation from F to G when F entails G but not vice versa. So, the set of 4 Aristotelian relations are "hybrid" according to the "pure" sets of logical relations. Demey and Smessaert therefore introduce two theoretical "pure" 2-D geometries: the "opposition geometry" (OG) and the "implication geometry" (IG), depicted in Figure 7 for 2-D systems:

(OG) Opposition relations:
contradiction (CD), contrariety (C), subcontrariety (SC), and non-contradiction (NCD);

(IG) Implication relations:
bi-implication (BI), left-implication (LI), right-implication (RI), and non-implication (NI).

The former inherits CD, C and SC from AG and replaces SA with the new relation of non-contradiction (NCD); the latter renames SA as left-implication (LI), and adds bi-, right- and non-implication (BI, RI, NI, respectively) as showed in Figure 7. In this way, it is possible to introduce a formal perspective on the "informativity" of the relations in OG and IG, based on the well-known idea of information as range: CD is the most informative relation in OG, NCD the least informative, and C and SC are in between; similarly, BI is most informative in IG, NI least informative, and LI and RI are in between. In this way, OG and IG jointly solve the hybrid problem of AG; furthermore, they have interesting historical precursors and exhibit a rich group-theoretical structure. In fact, OG and IG can be thought as two structures made of dual additive components. This 2-D geometrical approach can have an n-dimensional extension.

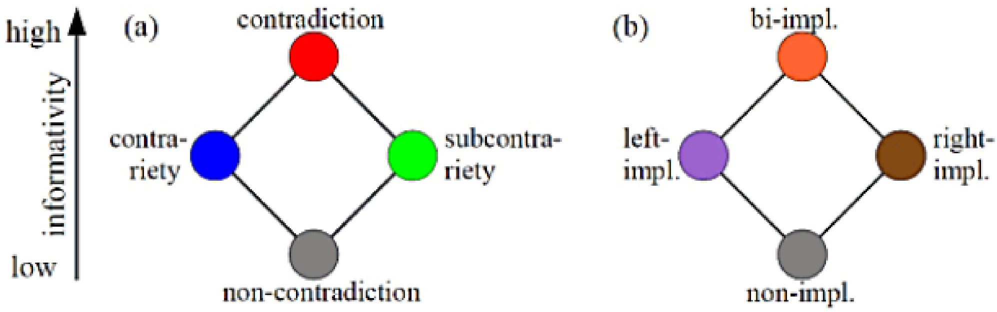

Figure 7: Informativity of the Opposition and Implication Relations [88].

According to CICT, the full information content of any symbolic representation (Domain or Application) emerges out from the capture of two fundamental coupled components: the linear one (unfolded, associated to external representation) and the nonlinear one (folded, associated to internal representation) (Figure 3). Therefore, to get full information conservation by E^2PM information processing, we need to take into consideration the linear representation and the non-linear one jointly. For the linear, external representation (additive duality), we can use the same Demey and Smessaert's approach for n-dimensional duality in Logic and Language. As a matter of fact, it can be based on two "pure" irreducible dual n-geometric subsystems to start with:

(NOG) n-HypercCube:
N-Opposition Geometry, N-Opposition Relations: Being True/False Together.

(NIG) n-HyperOctahedron:
N-Implication Geometry, N-Implication Relations: Truth Propagation,

which can be merged to form the RDH logic geometry.

To find the corresponding, coupled, non-linear, internal representation (multiplicative duality) of RDH, according to CICT, it is necessary to consider the geometric reciprocal of RDH, that is the cuboctahedron, henceforth referred to as COH, as reported in Figure 8. COH is somewhat special in being one of the nine edge-transitive convex polyhedra, the others being the rhombic dodecahedron, the five Platonic solids, the icosidodecahedron, and its dual, the rhombic triacontahedron.

The first appearance of COH is in the book titled “Archimedean Solids”, where Pappus of Alexandria lists solids and attributes to Archimedes in his Book V of his Collections, though Archimedes makes no mention of these solids in any of his works [73]. Long after, it reappears in Luca Pacioli’s book “De divina proportione” written around 1497 where all figures are drawn by Leonardo da Vinci [72]. Johannes Kepler (b.1571-d.1630) rediscovered the 13 Archimedean solids and gave the first surviving proof that there are only 13. In Japan cuboctahedra have been widely used as decorations in furniture and buildings. Lamps in the shape of cuboctahedra were used in Japan already in the 1200s, and they are still used today in certain religious ceremonies in memory of the dead [70].

In geometry, COH is named thusly because it is simply an intersection of a cube and an octahedron, as represented in the “Crystal” by M.C. Escher in 1947 [65]. A COH is a polyhedron with 8 triangular faces and 6 square faces (Figure 8). A COH has 12 identical vertices, with 2 triangles and 2 squares meeting at each, and 24 identical edges, each separating a triangle from a square. As such, it is a quasiregular polyhedron, i.e. an Archimedean solid that is not only vertex-transitive but also edge-transitive. The COH can be inscribed in the RDH and viceversa ([92], pp. 203-205.) The centers of the square faces determine an octahedron ([2], p. 137.)

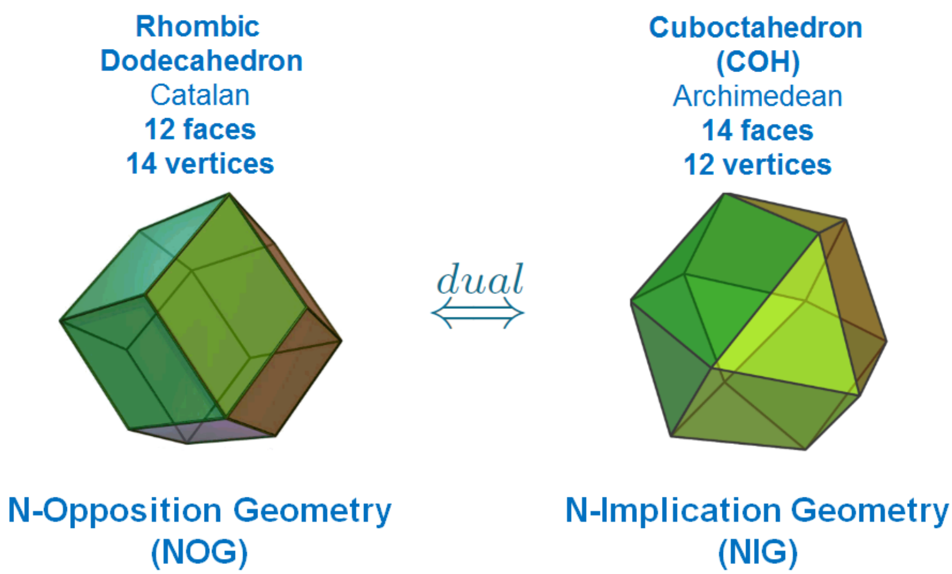

Figure 8: Complete Duality of Opposition and Implication Geometry in Logic Geometry and Language.

COH is at the center of American architect and systems theorist Richard Buckminster Fuller's (b.1895d.1983) synergetic philosophy in architecure. Fuller applied the name "Dymaxion" to this shape along with "Vector Equilibrium" (VE), meaning the dynamic balance of tensional cosmic forces, since, unlike Cartesian coordinate system, it can strikingly be developed around one nuclear central sphere (Figure 9) [40, 41]. The VE, as its name describes, is the only geometric form wherein all of the vectors are of equal length. This includes both the semidiagonals from its center point out to its circumferential vertices, and the edges (vectors) connecting all of those vertices. It was Buckminster Fuller who discovered the significance of the full vector symmetry in 1917 and called it the Vector Equilibrium in 1940. VE represents the ultimate and perfect condition wherein the movement of energy comes to a state of absolute equilibrium, and therefore absolute stillness and nothingness. The closest packing (positioning) of spheres (wave fronts) organizes in all of nature as a VE that is also the underlying "tensegral" form of the energetic "Torus", the core recurring pattern, "donut shaped energy vortex", that evolves life at every scale. As Fuller states: "The vector equilibrium nucleus of the isotropic vector matrix is the zero starting point for happenings or nonhappenings: it is the empty theater and empty Universe intercoordinatingly ready to accommodate any act and any audience" ([40], Sec. 503.031.) In other words it can be thought as the zero-phase energy point of quantum field theory, which all other forms emerge from (as well as all dynamic energy events). Theoretically, in quantum physics, the energy of the vacuum at zero degrees Kelvin is known as the zero point energy field. Various authors have given different names to this field of energy: The Field, Source Field, Morphic Field, Aether (or Ether), Akashic Field, etc. It is the vast sea of energy which propels the whole universe. You cannot actually observe the "VE" in the material world because it is the geometry of absolute dynamic balance. What we experience on Earth is always expanding toward and contracting away from absolute equilibrium. Like a wave arising from the surface of a tranquil sea, a material form is born (unfolds) from the plenum (fullness) of energy and dies (enfolds) back into it. The VE is like the imaginable, yet invisible, mother of all the shapes and symmetries we see in the world.

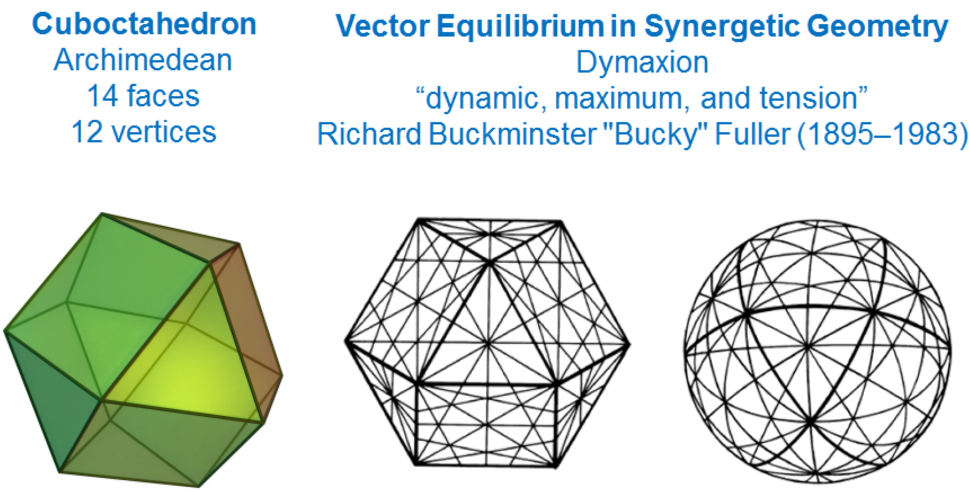

Figure 9: Cuboctahedron and the Vector Equilibrium in Synergetic Geometry, according to Richard Buckminster Fuller [40].

The most fundamental aspect of the VE to understand is that, being a geometry of absolute dynamic equilibrium wherein all fluctuation (and therefore differential) ceases, it is conceptually the geometry of what can be called the “zero-point” or “Unified Field”, also called the “vacuum of space”. In cubic close packing of equal spheres, each sphere is surrounded by 12 other spheres. Taking a collection of 13 such spheres gives the fundamental elementary cluster. Equilibrium of this kind is also called “isotropic vector matrix” (IVM) as an omnidirectional closest packing around a nucleus about which omnidirectional concentric closest packing of equal spheres form series of vector equilibria of progressively higher frequencies [41]. Connecting the centers of the external 12 spheres gives a COH [92].

The COH’s 12 vertices can represent the root vectors of the simple Lie group “$A3$”. With the addition of 6 vertices of the octahedron, these vertices represent the 18 root vectors of the simple Lie group “$B3$”. COH is described by a crystallographic “Cn” finite group theory and can play an essential role in the assembly of nanoparticle building blocks over multiple length scales into hierarchically ordered structures in the nonclassical crystallization pathway [97].

5 E^2PM Logic Geometry Universal Presence

Fom previous section, we saw that since ancient times thinkers, scientists or artists have been working on the artistic and scientific potentials of COH intuitively. More recently, the physicist Nassim Haramein in his Unified Field Theory suggests that the structure of spacetime has a COH "vector equilibrium" at its core. According to his theory, the structure can be seen in the close-packed hexagonal cells of honeycombs and bubbles, boiling water, and the storms on gas giants [44, 45]. COH is the ideal state of IVM. The IVM is formed by filling all space with cuboctahedra recursively.

Haramein's lifelong exploration into the geometry of space-time has resulted in a comprehensive unification theory based on a new solution to Einstein's Field Equations. Haramein's series of scientific papers [44, 45, 46, 47, 48] propose solutions to the long-sought quest for a Unified Field Theory, promising to revolutionize our current understanding of physics and our place in the universe.

At the base of his Unification theory is the COH. The 24 element octahedral group is denoted as "O" and is the set of all symmetries inscribed in S^2 (the ordinary 2-sphere in 3-dimensional Euclidean space), which is also the symmetry group of the cube since the eight faces of the octahedron correspond to the eight vertices of the cube. The 24 element group through S^2 yields the cuboctahedral group which we can relate to the U_4 (4-unitary group) space. The octahedral group O is isomorphic to the symmetric four group S^4 (the 4-sphere in 5-dimensional Euclidean space). In turn, the S^4 group is related to the U_4 topology and the cuboctahedral group relates to the GUT (Grand Unification Theory) [78].

According to Haramein, the characteristics of matter are an expression of a fundamental division of the vacuum. Along with Elizabeth Rauscher, he derived a scaling law whereby the structures we find within the Universe, from the Planck distance to the observable Universe as a whole, lay along a scale structured by the "phi ratio", also known as the "golden ratio" [49].

So, beyond the VE's primary zero-phase symmetry, the 64 Tetrahedron Grid, as it is known, represents the first conceptual fractal of structural wholeness in balanced integrity. It is noteworthy that the quantity of 64 is found in numerous systems in the cosmos, including the 64 codons in our DNA, the 64 hexagrams of the I Ching (Chinese Book of Changes) central in the teaching of Confucius (b.551-d.479 BC), the 64 tantric arts of the Kama Sutra, as well as in the Mayan Calendar's underlying structure. It appears that the 64-based quantitative value is of primary importance in the fundamental structure of the Unified Field and how that field manifests from its implicate (folded, premanifest) order to its explicate (unfolded, manifest) order, both physically and metaphysically.

The relationship of the finite and infinitesimal groups is key to understanding the symmetry relation of particles, matter, force fields or gauge fields and the structural topology of space, i.e., real, complex, and abstract spaces. In any case, the coupled n-RDH and n-COH logical geometry systems allow to achieve E^2PM n-dimensional full information conservation with universal pervasiveness. The interested reader to dig deeper into COH universal presence in art and science is referred to Kappraff [56].

6 Conclusion

According to CICT, to cope with ontological uncertainty effectively at system level, it is possible to use two coupled irreducible information management subsystems, based on the following ideal coupled irreducible and complementary asymptotic dichotomous concepts: "Information Reliable Predictability" and "Information Reliable Unpredictability" [36, 37]. In this way, to achieve realistic behavior, the overall system must guarantee both Logical Closure (Reactive Information Management, "to learn and prosper") and Logical Aperture (Proactive Information Management, "to survive and grow"), both fed by environmental "noise" [20, 28]. Thus, a natural operating point can emerge as a new transdisciplinary reality level, out of the interaction of two complementary and irreducible information management subsystems, interacting with their common environment.

Traditional EPM can be thought as a reliable starting subsystem (closed logic, operative management) to initialize a process of continuous self-organizing and self-logic learning refinement (open logic, strategic management subsystem), fundamental to achieve E^2PM system model. This ontological uncertainty management (OUM) method can capture natural logic dynamics behavior, as function of specific unpredictable interaction, unknown at system design level, according to available system resources.

Though the hypercube logic geometry seems to be a straight-forward method to depict the logic relations in propositional logic, further research must be planned to go beyond this first approach of the notation. Future studies ought to validate empirically the contribution of this logical geometry approach to the understanding of logical relationships, notably in educational settings. The intuitive character of the related algebra to apprehend logical relations must be tested in comparison with classical methods of learning.

Through the hypercube and COH logic geometry, we propose a notation that goes beyond a format distinction and constructed with the purpose to facilitate inferences either on a diagrammatic representation, or a lexical one. The latter particularly allows operations on complex propositions within hypercube with more than three dimensions, mentally difficult to imagine. This algebra, by posting directly configurations in which a complex proposition is true, can explicitly represent all mental models, in the sense of Johnson-Laird [54], necessary for the apprehension of a proposition in all its complexity. In agreement with Morineau [71], we think that this algebra could represent a tool for assisting work activities that involve inductive reasoning, like problem- and case-based reasoning in medical diagnosis [27], and subject profiling in psychiatry and psychotherapy [11, 50, 52]. More specifically, from a biomedical engineering perspective, fault diagnosis task [79] and troubleshooting on logical networks [82] could be areas of application for reliable testing and validation of the presented EPM extension as "Evolutive Elementary Pragmatic Model" (E^2PM).

According to Matte Blanco, every human psychic phenomenon turns out to be a bilogic process which is a chain of symmetric and asymmetric subprocesses whose combination modes are, a priori, various and infinite, giving rise to the rich variety of human thoughts. According to CICT, the coupled n-RDH and n-COH logic geometries can model quite closely Matte Blanco's concept and beyond, according to available computational resources. In any case, the combination of the double bind theory by Bateson (DBT) [3] and Matte Blanco's bilogic [67] provides concrete opportunity for the foundation of computational psychiatry and for reformulating psychoanalytical theory in a shareable way, according to Arden early vision, anticipated in 1984 [1].

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