

The Works about Logic of Francisco Miró Quesada Cantuarias

Luis Adolfo Piscoya Hermoza

Abstract

This is a critical presentation of the works written by Francisco Miró Quesada Cantuarias on logic and related subjects. We consider that Miró Quesada's contributions in this area can be divided in three subareas: (a) his work as a teacher of logic, (b) his studies on the philosophy of logic and mathematics, and (c) his theoretical studies of rationality or reason. We discuss his contributions and provide critical remarks.

Keywords: Peruvian logic, teaching of logic, theory of reason.

Introduction

The late philosopher Francisco Miró Quesada Cantuarias (1918–2019) was perhaps the most important philosopher in Peruvian history, and undoubtedly our most important philosopher of logic of the last century. In this article, we make a critical presentation of his works on logic and philosophy of logic. The size and quality of his production in these areas, however, force us to resort to some kind of structure to organize our exposition. Such a structure, of course, which will naturally be subject to risky limitations.

We propose to distinguish three aspects of Miró Quesada's logical work: first, his activity as an educator specialized in logic; second, his activity as a philosopher of logic and mathematics; and, finally, his activity as an analytical philosopher in the construction of a general theory of reason¹. These aspects were not developed independently in Miró Quesada's work but are connected with each other as well as with other aspects of his intellectual production that are beyond the scope of our exposition. But it must be said that, in general,

¹This last area encompasses other logic-related works like his pioneering works on legal logic [7, 8, 12], but which we will not be able to talk about here. However, this is treated in C. A. Serbena's contribution to this volume [27].

the different aspects of the intellectual activity of our philosopher display an impressive unity and coherence.

The last aspect we pointed out—that of his activity as a theorist of reason—is the one upon which Paco’s epistemological thought was grounded. Throughout his vast work, our author was *primarily* concerned with grounding rational knowledge by investigating what he has frequently called the ‘structure of reason’. It was from this concern that Paco developed his work as a philosopher of logic and of mathematics, as well as of law, of politics, and of morals. Miró Quesada argued that, if we were able to clarify the internal structure of reason and the principles that regulate its functioning, then we would be able to explain the nature of knowledge and of rational activity as a whole.

For Paco, the existence of alternative geometries and the emergence of ‘heterodox logics’, as he called them [15], does not affect the viability of this hypothesis. Rather, it may lead us to the discovery of a unitary system of reason capable of explaining the diverse logical and mathematical systems as “variations of the same melody” [15, p. 41].

Although discussing whether Miró Quesada’s philosophical thinking is in line with this project goes far beyond the scope of this presentation, we consider it appropriate to contextualize our presentation of his works within his philosophical project. It is from this context that we will be able to refer to what, in our opinion, constitute some possible corroborators of the conjecture guiding his overall thought, as we understand it.

1 The Teacher of Logic

Miró Quesada was one of the first university professors to write a textbook mathematical logic in Spanish: his university course of logic [5]. This text is possibly the first textbook of mathematical logic produced in Latin America, having as its only predecessor in Spanish language—as far as we have been able to trace—a book by Spanish professor Juan David García Bacca [4].

The publication of this course meant the beginning of serious and systematic studies of this subject [e.g. 3, 25] as well as the foundation for the development of a rigorous studies of epistemology and philosophy in Peru; in particular, of a philosophy that was sensitive to the new challenges posed by a society that was already becoming a techno-scientific society.

This publication was followed by other textbook in which first-order languages and their application to the formalization of set theory [9] and the reformulation of classical Aristotelian logic stood out. This persistent and dedicated work had its most notable landmark in the book *Lógica 1: Filosofía de las Matemáticas (Logic 1: Philosophy of Mathematics, 1980)* [17]. Having a considerable breadth and density, this volume provided a pedagogical treat-

ment quite unusual in logic books, which is an expression of the educational vocation that has marked Paco's university work. *Lógica 1* was for a long time the only serious, rigorous, up-to-date, and well-founded university text on mathematical logic ever published in Peru. Among its virtues, we can point out an introduction to model theory of such a generality that includes non-enumerable infinite sets, and the presentation of a system of natural deduction that dispenses with open formulas.

In addition to his production of rigorous texts on logic and his disciplined and passionate teaching, Paco was also active as a nationwide promoter of innovation and improvement in logic studies. We owe to him the introduction of an elementary course of logic in the official programs of Peruvian secondary education. Furthermore, he himself led the training of the first generation of teachers to assume this task and was the author the first Peruvian school textbook on logic [6], whose successive re-editions continued to be used for several decades.

2 The Philosopher of Logic and Mathematics

Although Paco's philosophy of logic is closely linked to his central hypotheses on human reason, we have decided to treat these two topics separately, using as a demarcation criterion his emphasis on what we may consider traditional topics in pure logic, as well as in the philosophy of logic and mathematics.

One of the recurring concerns in Miró Quesada's writings on logic is the distance, and even divorce, existing between our intuitions or fundamental evidences and some properties of the existing logical systems or languages. With regard to the standard deductive procedures, he was puzzled by the fact that a deductive chain, where the truth of the conclusion is supposed to be derived from that of the premises, often includes intermediate links constituted by open formulas. What Miró Quesada finds disturbing is that such open formulas, taken in isolation, are not interpretable as propositions and, therefore, cannot be said to be either true or false. This would contrast with one of our basic intuitions that conceives a sound deductive chain as a sequence of true propositions such that each of those propositions, excluding the premises, is a logical consequence of the previous ones.

To overcome this difficulty, Paco proposed a system of first-order logic without variables [14] which is standard in all but two respects: (1) it introduces the notion of a letter scheme, which can be substituted by any individual constant to construct ordinary axiomatic schemes; and (2) it introduces a system of subscripts to dispense with bound variables. This would avoid the intermediate open formulas of traditional deduction systems. At the same time, it would operationally prove that the concept of individual variable is unnec-

essary for constructing a natural deduction system, since a deduction system without variables would be compatible with our intuitions. Considering this, Paco attributes the counter-intuitive or unnatural distortions of deduction to the fact that logic borrowed from mathematics the notion of variable, which does not exactly fit its needs.

From a more semantic than syntactic perspective, Paco has been concerned with the fact that, intuitively, the notion of logical entailment² presupposes or suggests a relation of ‘atingence’ or connection between the antecedent and the consequent. However, in standard logic systems it is possible to construct a set of formulas that conflict with this relation of atingence, and which constitute what are commonly known as the paradoxes of material implication. Moreover, similarly to some notable researchers, our author has paid special attention to the way in which these paradoxes affect Tarski’s definition of the concept of logical consequence and has noted that this definition is also limited by the presence of modal logics, polyvalent logics, and the systems to which he has given the name ‘paraconsistent’ [cf. 23], which generally have a more complex semantics than that of bivalent systems.

Drawing on the well-known research of Anderson and Belnap [1], Miró Quesada has resorted to the concept of logical relevance [19] to overcome the paradoxes of material implication and Lewis’s strict implication. With the help of this notion, he determined some necessary and sufficient conditions of logicity leading to a concept of logical consequence that is more compatible with our intuitions. Relevance logic, then, helped Paco’s search for formalisms better suited to our intuitions, even if this forces us to take up again the notions of essence and intellectual intuition that were dismissed by positivists and formalists. In this effort, he warns us that, although the systems of relevance logic in use constitute a step forward in logic, they are insufficient because they are based on a somewhat imprecise concept of relevance that functions as a necessary but not sufficient condition of logicity.

Within this subject, Miró Quesada has devoted more than one study to the concept of logical consequence. These studies were motivated not only by the divorce between formalization and intuition, but also by the need to achieve the greater generality that is required by the highly diversified growth of logical systems, which together already exceed the scope of a definition that is about one century old.

His inquiries in this direction have led him to propose what he has called *transmissive logic* [16], which provides him with a framework for constructing a definition of logical consequence whose generality would allow him to deduce, as a particular case, that of Tarski. To achieve this goal, he replaces the classical

²When translating ‘entailment’ into Spanish, Paco often used the word ‘entrañamiento’, an expression that may be literally retranslated into in English as ‘entrailment’.

truth values by the concepts of signed and anti-signed value, which are then generalized by means of the concept of *signedness*. In this way, he understands the relation of logical consequence as a relation that transmits some signed value from antecedent to consequent. In the specific case of bivalent logic, this signed value is ‘true’. He also points out that it is possible to extend this definition to cases that include relations between signed and anti-signed values, but he prefers to restrict his proposal only to the transmission of signed values, which he typifies as homogeneous transmission. Thus, he proposes, as a first result of his research, to define the expression ‘ A entails B ’ or ‘ B is a logical consequence of A ’ as equivalent to the situation where the conditional $A \rightarrow B$ establishes a relation of homogeneous transmission.

This proposal seems to Miró Quesada to be in line with our intuitions because it coincides with that of Tarski in the bivalent case, but it would be exempt from the inconveniences raised by the paradoxes of material and strict implications. Moreover, given its generality, it could be used in polyvalent logics (classical or paraconsistent) and in modal logics. An additional advantage is that it would allow a more general treatment of model theory. However, a limitation noted by Paco himself is that his proposal does not include languages with negation. For us, this means that his theory was left in its initial stage, and—to the best of our knowledge—he did was not able to further develop it.

In the field of the philosophy of logic and mathematics, a permanent concern of Miró Quesada was the ontology of mathematical objects and logical principles. His persistence in this problem is understandable inasmuch as he has considered that unveiling the ontological status of the logical principles would provide an answer of substantial significance regarding the nature of reason and rational knowledge. Paco has thoroughly discussed and analyzed the formalist theses of a neo-positivist style, as well as those by intuitionists and Neo-Platonists. He has also formulated interesting and sharp objections to these proposals (which today we can call traditional) and has distanced himself from them, although not to the similar extents.

Already in his book *Apuntes para una Teoría de la Razón* (*Notes for a Theory of Reason*, 1963) [10], Miró Quesada sought to neutralize formalism by resorting to Gödel’s incompleteness theorem. This theorem, as is well known, establishes that any formalization of a theory of a complexity equal to or greater than that of Peano’s arithmetic is necessarily incomplete insofar as it is always possible to construct, with its own language, a true proposition that is neither deducible nor rejectable as a theorem of it. Likewise, it is not possible to construct a consistency proof for such a theory using the expressive means (i.e., the language) of the theory itself. In short, we have that all members of a very important family of mathematical theories, which includes Zermelo-Fraenkel’s set theory, are in principle limited in proving their own consistency.

It is worth noting here that Gödel's theorem (along with the proliferation of non-classical logics) has been one of the main sources of insight for Miró Quesada's general theory of reason. His reflections on this theorem yielded many fruits for his philosophical conceptions of logic and mathematics, including his identification of the principle of surpassing that we will discuss later.

In the aforementioned *Apuntes*, as well as in other works, Paco has argued that these results prove conclusively the limitations of formalisms and show the impossibility of Hilbert's old project. He stresses that the truth of a proposition exceeds the deductive possibilities of formalisms and that, consequently, their ground must be sought outside them. In this direction, he formulates the thesis that propositions such as the one constructed by Gödel in the proof of his theorem (those that are true but deductively undecidable) are genuine examples of Kantian a priori synthetic judgments [13]. In this way, he takes up an aspect that is dominant throughout his philosophical thought, sometimes explicitly and sometimes implicitly.

In relation to intuitionism, Miró Quesada views with enthusiasm the constructive method, to which he recognizes its almost empirical character and great reliability expressed in the exclusive use of recursive functions. However, he also warns that any constructive procedure presupposes the existence of a minimum set of irreducible concepts which, therefore, are not in turn constructible. He points out that the only way left for intuitionists like Brouwer to ground their primitive concepts is to refer to the criterion of evidence, which brings serious difficulties. Thus, for example, the rejection of some logical principles that for a long time were considered evident, as is the case of the principle of the excluded third, has led intuitionists to reject the so-called proofs by *reductio ad absurdum* and, therefore, to a project of total reconstruction of mathematics that has been defeated in fact by the theoretical difficulties it generated and, finally, abandoned.

Miro Quesada has maintained, in his *Apuntes* and in later works [19], that the historical development of knowledge shows the existence of authentic evidence and inauthentic evidence. This situation would require a criterion or a methodology to distinguish between them, a resource that intuitionists have not shown to possess. His analysis has led him to maintain that such a distinction is not possible if the concepts or principles that are postulated as self-evident are not formalized. For Miró Quesada it is precisely formalisms (such as those proposed by Frege, Russell, and their successors) what has made it possible to detect the limitations of some axioms and postulates of Euclid or of principles (such as that of the excluded third) that were considered, for centuries, to be self-evident. Thus, inauthentic evidence seems inevitably provisional and subject to what he preferred to call the *principle of surpassing*, a principle of a predominantly meta-theoretical nature by which the capacity of reason to

surpasses or exceeds its own deductive consequences.³ Gödel's incompleteness theorem would be an example that illustrates how this principle operates, for in it we see how the set of theorems we can prove from any formalization of Peano's basic arithmetic is exceeded or surpassed by the set of all the theorems that this theory has.

Vague intuitions such as those of dialectical logic are also subject to this principle. In spite of the opposition with formal logic insisted upon by several defenders of dialectical logic, the intuition of a dialectical logic could only be rendered precise by its formalization through paraconsistent languages such as those of Routley [26]. It was thus that the vague notion of dialectical logic was surpassed by formal logic—when it was extended into the realm of heterodox logics—making it possible to articulate a formal dialectical logic. However, the very act of surpassing does not weaken reason, but rather affirms it insofar as it is applied under certain universal and necessary principles postulated by Miró Quesada. Such a view places Paco in a position that seems to share some weaknesses with mathematical intuitionists, even though it has been the object of their criticisms.

Regarding his own stance, a certain underlying enthusiasm for Platonism is detectable in Paco's writings, although not an explicit adherence. This has never surprised me since, as a student, I once heard him say in class that Plato was the greatest philosopher of all times and the one responsible for the most enigmatic and enduring challenges.

In his article on Rieger's objection and the horizon of mathematical ontology [11], for example, he defends Gödel's Platonism, but he remarks that this point of view suffers from limitations due to misinterpretation of the Löwenheim-Skolem theorem, which establishes the non-categoricity of Peano arithmetic as formalized by a first-order language. According to Miró Quesada, Rieger intends to use the results of this theorem to object to Gödel's Platonism. According to Rieger, the existence—proved by the aforementioned theorem—of non-standard models that satisfy the definition of number expressed by Peano's axioms is proof that the idea of natural number can be understood in several ways and, therefore, cannot be Platonic. Paco considers, however, that Rieger has interpreted things backwards, since what the proof of the non-categoricity of Peano's axioms proves is the insufficiency of formalisms to univocally characterize the idea of natural number. Therefore, it would rather be an objection

³We translate as 'principle of surpassing' what Paco called in Spanish 'principio de rebasamiento', in keeping with his use of the term 'surpass' in one of his publications in English [22, pp. 641, 644, 646]. The main sense of the Spanish verb 'rebasar' is 'to pass or exceed a certain limit', although Paco also uses this term in its sense of 'supersede'. Whenever we use the term 'surpass' (either as a verb or as a noun) in this paper, we are doing it in the technical sense related to this principle.

to the formalist thesis that would leave the Platonic proposal as an alternative.

Miró Quesada concedes nevertheless that the consequences of the Löwenheim-Skolem theorem are contrary to Platonism insofar as they engender a paradox that can only be solved if we relativize certain mathematical properties to those of the languages that express them; which, by the way, shows the difference between mathematics and formal mathematical languages. The referred theorem, as is well known, states that every first-order theory satisfiable by a model is also satisfiable by a denumerable model. Since set theory is a satisfiable first-order theory containing statements about the set of real numbers, then it must be satisfiable by a denumerable model according to the above theorem. But this would contradict the result of Cantor's diagonalization theorem, which states that the set of real numbers is not denumerable. According to Paco, this difficulty can be explained by attributing it to the differences in the linguistic systems used by Löwenheim and Skolem, on the one hand, and those used by Cantor, on the other hand.

3 The Theorist of Reason

The most original and sustained side of Paco's main philosophical project is, undoubtedly, his unwavering determination to achieve a satisfactory explanation of rational knowledge by unveiling the structure of reason and the invariant principles that regulate its functioning. This ambition constitutes both a thesis and a research program to which he was devoted throughout his life, at least since the early sixties with the publication of his *Apuntes*.

The sense of the previous proposal is of ostensible Kantian inspiration, since, throughout his many works, Miró Quesada has maintained that rational knowledge is only possible if it can be formulated in terms of propositions that are universally and necessarily true. He has expressed this point of view even more radically in his later works [e.g. 21] by characterizing any effort to demonstrate the impossibility of universally and necessarily true statements as an impure use of reason. Furthermore, he typified as skeptical anyone who makes impure use of reason, Ludwig Wittgenstein being characterized by Miró Quesada as the most illustrious of the contemporary skeptics for proposing the relativization of the properties of propositions and of language as a whole to a given *language-game*. Thus, the core of the philosophical discussion is given, according to Miró Quesada, by the classical controversy between rationalism and skepticism.

In short, Paco argues that, despite the remarkable arguments in favor of skepticism, such as Wittgenstein's, and the proliferation of alternative logics that omit or transform the use of principles traditionally conceived as unshakable, it is possible to detect or discover, in each case and with ingenuity,

invariant rational principles whose validity is presupposed by all intelligible argumentation, including that of the skeptic. One of these supreme principles is, according to Miró Quesada, the principle of non-contradiction, which would be a kind of censorship measure, so to speak, against the unpredictable results that the exercise of reason can produce. Likewise, this unpredictability of the products of reason is what clearly separates the rational principles from the rules of the game whose effects are always foreseeable insofar as they can be stipulated by the players. Not considering this difference, according to Miró Quesada, was Wittgenstein's cardinal error in the *Philosophical Investigations*.

According to the above point of view, sustained by Miró Quesada in the whole of his written work as far as we know it, the principle of non-contradiction is essentially unaffected by paraconsistent logics because they do not suppress it, but only weaken its deductive use. Thus, while a formula $A \wedge \neg A$ could no longer trivialize a theory formalized by means of a paraconsistent logic such as da Costa's C_1 system [2], it could be so if the negation of such a formula were made in terms of the strong negation (\neg^*) of that same system. Thus, the formula $A \wedge \neg^* A$ could trivialize a family of finitely trivializable paraconsistent systems. These results would support the validity of the principle of non-contradiction, which, more than a rule of logical calculus, is a kind of principle of intelligibility, insofar as, for example, we could not understand someone who simultaneously affirms that the system S is and is not paraconsistent.

There are, according to Miró Quesada, other similar principles that research will discover, among which he glimpses the principle of surpassing that we have mentioned in the previous section, of which dialectical synthesis is a particular case. This principle is not only fulfilled in pure logical-mathematical contexts, as we have seen with Gödel's incompleteness theorem, but also in the relations between theory and reality and, hence, in the phenomenon of the ideologization of theories [cf. 18]. We know that a set of true propositions about a certain section of reality can be deduced, with the same logical validity, from diverse sets of hypotheses, and that nothing in the rules of deduction forbids that some these sets of hypotheses be false. In situations like this, thus, it is possible to choose as explanatory context a set of hypotheses, regardless of their truth or falsity, that is most compatible with certain interests, that is, a set of hypotheses more closely aligned with a given ideology. This would lead to reality only partially coinciding with this ideologized theory and, consequently, to reality surpassing that theory. Certainly, no contradiction is required for the surpassing to take place, which distinguishes surpassing from dialectical synthesis and shows the greater generality of the former over the latter.

The touchstone of Paco's theory of reason is the recognition that expressions such as 'properly grounded knowledge', 'rational validity', or 'rational principle' can only be adequately understood if we assume the existence of universal and

necessary (i.e., absolute) rational knowledge or principles that would be, always according to his view, the condition of possibility of philosophy as such [20]. It is in this sense that reason is defined as the faculty of the universal.

In later works, Miró Quesada adds to the principles of non-contradiction and of surpassing others that correspond to some of the rules of deduction typical of propositional logic. These additions, in our view, do not match our author's explicit recognition of the legitimacy of heterodox logics, although developing a counterargument in this regard would largely exceed the purposes of this exposition. What matters in this case is that, even if we admit by hypothesis the existence of an invariant core of absolute principles that can justify rational knowledge, this does not solve the crucial problem posed by the way in which they are constituted. Paco was aware of this limitation, although it did not prevent him from recognizing the merits of Piaget's research in this direction [e.g. 24], emphasizing that it is the best we have at hand for understanding the psychogenesis of logical and mathematical concepts. He stresses nevertheless that these attempts are insufficient to understand, for example, the process of acquisition of the notion of the transfinite cardinal.

Faced with the above difficulties, our author cannot but ground the principles he postulates as universal and necessary to the classical intellectual intuition capable of grasping authentic evidence and the essential properties of reality. (Incidentally, Miró Quesada devotes the ninth chapter of his *Apuntes* to the question of intellectual intuition.) He admits that this may seem objectionable to positivists and other like-minded authors, but he still finds it more satisfactory than pragmatist, empiricist, and Wittgensteinian solutions. However, this return to essentialism, which he does not perceive as a vicious circle, seems to place the question very close to where it was left by Plato and Kant, philosophers with a seemingly defining influence on Paco's thought.

4 Conclusion

The logical work of Miró Quesada is, no doubt, a prolific work with a remarkable repercussion in the Latin-American and worldwide academic community. However, we do not wish to close this exposition without pointing out what we consider to be those areas where it is most vulnerable to rigorous criticism.

Our first remark is related to his project to identify universally and necessarily true principles that can justify rational knowledge and his demand that any genuinely philosophical justification must be rational in this sense. Such a project, it seems to us, brings the discussion back to the metaphysical sphere in which it traditionally took place. With this, our thinker would place himself in a scenario that had already been rejected by his logical-rigorous vocation in various parts of his work. We consider that this thesis is weak because it

implies the existence of true rational principles independently of any linguistic and historical context, which conflicts with Tarski's undefinability theorem.

According to this theorem, we can give a definition of a true proposition only within a language of definite structure, being impossible to give a truth definition that is adequate for all languages. One can certainly grant more liberal uses of the predicate 'true', such as the one Paco seemed to have in mind. But this will confront him with the fact that Tarski's theorem presupposes the principle of non-contradiction and of other logical rules. We say this because Miró Quesada explicitly advocated for the legitimacy of anomic heterodox logics, where those principles are banned or at least restricted.

Secondly, it seems that the sense in which Paco uses the concept 'universal' is not always the same. In a first sense, we can consider that a proposition is universally true when it is true for all the objects it refers to within a linguistic context. In a second sense, a proposition is universally true when it is valid for every possible interpretation assigned to it, which liberalizes the semantic context, but not the syntactic one. The principle of mathematical induction, for example, is universally true in the first sense but not in the second one, which is used to demonstrate that this principle is independent of the other axioms of Peano. Miró Quesada does not specify whether he is using one of these senses of 'universal', both, or another one. In our estimation, however, either sense is equally unfavorable to his project, inasmuch as they only allow us to speak rigorously of the truth of a proposition within a context.

Third, I consider that Miró Quesada does not prove convincingly his thesis that *rationally grounded* knowledge is but knowledge *grounded on universally and necessarily true principles*. Our author dismisses the possibility of understanding *rationally grounded knowledge* in terms of *knowledge obtained through the best method available to us at a given historical moment*, unless the nature of that 'best method' is to be decided a priori. The Kantian and/or Platonic apriorism underlying this thesis has received hard historical blows, and in his own account of apriorism, Paco cannot but recourse to some sort of metaphysical intuition far removed from the more properly scientific practice of proposing formulations delimited in scope.

Our last remark is related to his proposal of a first-order deductive system without variables. It seems to us that, given its long list of stipulations, this system is not simpler than the usual ones that do not use the rule of substitution. Moreover, in order to achieve the goal that each intermediate deductive step be a proposition, this system requires us to assume, by hypothesis, that non-quantified schematic formulas are propositions. This leads to the inconvenient situation of giving schematic letters α, β, \dots the same status as constants. As for his proposal of a generalized definition of the concept of logical consequence, I believe that it can be judged and evaluated more objectively if it is

effectively extended into languages with negation, which is a task that Paco bequeathed to future generations of logicians interested in developing his work.

None of these objections, however, diminishes the value of Miró Quesada's colossal intellectual enterprise, but rather make it a source of fruitful controversies on topics as important as they are difficult. Even those of his approaches to these controversies that we consider more erroneous also seem to us to be examples of philosophical reflection from which the next generations of philosophers of logic, mathematics, and reason will have to work in order to achieve more refined and better argued points of view.

Acknowledgments

I would like to thank Luis Felipe Bartolo Alegre and José Carlos Cifuentes for their valuable remarks to this paper.

References

- [1] A. R. Anderson and N. D. Belnap. *Entailment: The Logic of Relevance and Necessity*, Vol. I. Princeton University Press, 1975.
- [2] N. C. A. da Costa. On the theory of inconsistent formal systems. *Notre Dame Journal of Formal Logic*, 15(4):497–510, 1974. DOI: 10.1305/ndjfl/1093891487.
- [3] J. B. Ferro Porcile. *Procedimientos Decisorios para Fórmulas Monádicas de Primer Grado (Decision Procedures for First-Degree Monadic Formulas*, in Spanish). PhD thesis, Facultad de Letras y Ciencias Humanas, UNMSM, Lima, 1966. URL: <https://repositorio.unmsm.edu.pe/handle/UNMSM/1895>.
- [4] J. D. García Bacca. *Introducción a la Lógica Moderna (Introduction to Modern Logic*, in Spanish). Labor, 1936.
- [5] F. Miró Quesada Cantuarias. *Lógica (Logic*, in Spanish), volume 6 of *Biblioteca de la SPF*. SPF, 1946.
- [6] F. Miró Quesada Cantuarias. *Lógica: Curso Escolar (Logic: School Course*, in Spanish). Miranda, 1948.
- [7] F. Miró Quesada Cantuarias. Problemas fundamentales de la lógica jurídica (Fundamental problems of legal logic, in Spanish). Master's thesis, Facultad de Derecho, UNMSM, Lima, 1953.

- [8] F. Miró Quesada Cantuarias. Teoría de la deducción jurídica (Theory of legal deduction, in Spanish). *Diánoia*, 1(1):261–91, 1955.
- [9] F. Miró Quesada Cantuarias. *Introducción a la Lógica y a la Teoría de los Conjuntos* (Introduction to Logic and Set Theory, in Spanish). Instituto para la Promoción de la Enseñanza de las Matemáticas, 1962.
- [10] F. Miró Quesada Cantuarias. *Apuntes para una Teoría de la Razón* (Notes for a Theory of Reason, in Spanish). UNMSM, 1963.
- [11] F. Miró Quesada Cantuarias. La objeción de Rieger y el horizonte de la ontología matemática (Rieger’s objection and the horizon of mathematical ontology, in Spanish). *Crítica*, 9(5):57–70, 1968.
- [12] F. Miró Quesada Cantuarias. La lógica del deber ser y su eliminabilidad (The logic of ought-to-be and its eliminability, in Spanish). *Derecho*, (30): 153–159, 1972.
- [13] F. Miró Quesada Cantuarias. Sobre los juicios sintéticos a priori (On synthetic a priori judgments, in Spanish). In *Actas del II Congreso Nacional de Filosofía*, Vol. 2: *Simposios*, pp. 260–265. Sudamericana, 1973.
- [14] F. Miró Quesada Cantuarias. Lógica de primer orden sin variables (First-order logic without variables, in Spanish). In D. Rosales Papa (ed.), *Lógica*, pp. 29–47. PUCP, 1976.
- [15] F. Miró Quesada Cantuarias. Las lógicas heterodoxas y el problema de la unidad de la lógica (Heterodox logics and the problem of the unity of logic, in Spanish). In D. Rosales (ed.), *Lógica. Aspectos Formales y Filosóficos*, pp. 13–44. PUCP, 1978.
- [16] F. Miró Quesada Cantuarias. Lógica transmisiva (Transmissive logic, in Spanish). Unpublished work presented to the IIF at the ULIMA, 1978.
- [17] F. Miró Quesada Cantuarias. *Lógica 1: Filosofía de las Matemáticas* (Logic 1: Philosophy of Mathematics, in Spanish). I. Prado, 1980.
- [18] F. Miró Quesada Cantuarias. Conocimiento científico, dialéctica e ideología (Scientific knowledge, dialectics and ideology, in Spanish). In *Filosofía y Ciencias sociales. Problemas Teóricos y Metodológicos*, volume 6 of *Temas y Problemas*, pp. 9–50. UC, 1982.
- [19] F. Miró Quesada Cantuarias. Sobre el concepto de relevancia lógica: El surgimiento de la lógica moderna y el abismo de la deducción (On the concept of logical relevance: The rise of modern logic

- and the abyss of deduction, in Spanish). *Archivos de la SPF*, 5: 64–79, 1985. URL: <https://sociedadperuanadefilosofia.org.pe/wp-content/uploads/2022/05/Archivos-V-envio-64-79.pdf>.
- [20] F. Miró Quesada Cantuarias. Historicismo y universalismo en la filosofía (Historicism and universalism in philosophy, in Spanish). *Archivos de la SPF*, 6:67–89, 1988. URL: <https://sociedadperuanadefilosofia.org.pe/wp-content/uploads/2022/05/Archivos-VI-envio-69-91.pdf>.
- [21] F. Miró Quesada Cantuarias. Crítica de la razón impura (Critique of impure reason, in Spanish). pp. 13–29. UNMSM, 1988.
- [22] F. Miró Quesada Cantuarias. Paraconsistent logic: Some philosophical issues. In N. J. Priest G., Routley R. (ed.), *Paraconsistent Logic. Essays on the Inconsistent*, pp. 627–652. Philosophia, 1990.
- [23] F. Miró Quesada Cantuarias. In the name of paraconsistency. *South American Journal of Logic*, 6(2):163–171, 2020. Annotated and translated by L. F. Bartolo Alegre.
- [24] J. Piaget. *Genetic Epistemology*. The Norton Library, 1970.
- [25] L. A. Piscoya Hermoza. *La Lógica Subyacente en el Principio de la Inducción Matemática (The Logic Underlying the Principle of Mathematical Induction, in Spanish)*. PhD thesis, Facultad de Letras y Ciencias Humanas, UNMSM, Lima, 1976. URL: <https://hdl.handle.net/20.500.12672/16656>.
- [26] R. Routley and R. K. Meyer. Dialectical logic, classical logic, and the consistency of the world. *Studies in Soviet Thought*, 16:1–25, 1976. DOI: 10.1007/BF00832085.
- [27] C. A. Serbena. Notas sobre a lógica jurídica de Francisco Miró Quesada Cantuarias (Notes on Francisco Miró Quesada Cantuarias’ legal logic, in Portuguese). *South American Journal of Logic*, 6(2):223–236, 2020.

Luis Adolfo Piscoya Hermoza
Facultad de Letras y Ciencias Humanas
Universidad Nacional Mayor de San Marcos (UNMSM)
Av. Universitaria cruce con Av. Venezuela cuadra 34, Lima, Perú
E-mail: lpiscoyah@unmsm.edu.pe