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Lewis Carroll on Some Fundamental Concepts in Logic

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Abstract

This is a survey a survey and analysis of seven problems of conceptual clarity that were faced, sometimes unsuccessfully by Lewis Carroll in his *Symbolic Logic* and other writings on logic.

Keywords: Lewis Carroll, singular terms, quantifiers, negation, existential import, logical copulae.

Charles Dodgson (1832-1898), better known as Lewis Carroll, was a mathematician whose entire career was spent at Oxford. Though he maintained a growing interest in logic, he was something of an outsider among his contemporary algebraic logicians (e.g., Boole, DeMorgan, Venn, Jevons, etc.). However, he did engage in extensive correspondence with many of them as well as a number of philosopher-logicians. His published works on logic appeared in the final decade of his life. Most important among these was *Symbolic Logic, Part* I(substantial fragments of two further parts were later discovered and published by W.W. Bartley in 1977). While Carroll made interesting and even important contributions to the field of logic (especially logical notation, diagrams, semantic tableaux, and his famous paradox of inference, he, like many others, often struggled to find a clear understanding of certain fundamental logical concepts. What follow here is a brief attempt to analyze and shed a bit of light on what he wrote about some of these.

1 "A Very Subtle Difficulty"

In later editions of *Symbolic Logic, Part I*, Carroll introduced (in Book II, Chapter II, Propositions of Existence) a new account of the logic of *existing*

things that had generated a "very subtle difficulty" in the first edition. In 1957, reviewing *Symbolic Logic*, the philosopher-logician Arthur Prior (1914-1969) appears to be the first to have noted the import of such a change. He wrote:

[I]n earlier editions Carroll gives as the "normal forms" (his own phrase) of his two sorts of existential propositions "Some S's exist" and "No S's exist." Now, however, the normal forms are "Some existing things are P" and "No existing things are P's." By this alteration, he claims, he is "evading a very subtle difficulty which besets the other form"; meaning no doubt that the first form suggests, as the second does not, that S's are in principle divisible into ones that exist and ones that do not. [13, 310]

It seems that Carroll saw what he took to be a good reason for changing his mind about propositions such as 'Some honest men exist'. Having first analyzed this as (1) 'Some honest men are existing things', he then found a "very subtle difficulty" requiring a new analysis: (2) 'Some existing things are honest men'. "Subtle" indeed. One's initial reaction would be that the two versions are logically equivalent (by simple conversion). Prior saw that any proposition of the form 'Some X is ..' can be construed as either referring to the X things that exist (= are in the universe of discourse) or to the X things that exist or don't exist. Carroll took this to be the problem of saying what a thing is, where 'thing' is understood as 'in the universe of discourse'. So now consider again the two analyses. The subject of (1) is 'Some honest men'; the subject of (2) is 'Some existing things'. (2), unlike (1), wears its existential status on its face. The subject of (1) amounts to 'Some things that are honest men' (for Carroll) or 'Some existing honest men' (for Prior). In either case, the resulting analysis would be (1c) 'Some things that are honest men are honest men' or (1p) 'Some existing honest men are existing things'. Carroll was right to make the change. While subject-terms and predicate-terms might exchange their roles in a proposition (e.g., by application of conversion), subjects and predicates can't do so.

2 "The Bewildering Question"

In the prefaces to the second, third, and fourth editions of *Symbolic Logic*, Carroll ended his brief preview of the change he made from the first edition concerning his evasion of his "very subtle difficulty" with reference to such "subtle difficulties that seem to lie at the root of every Tree of Knowledge." He added that "they are *far* more difficult to grapple with than any that occur in its higher branches." Carroll concluded that even the most foundational of difficulties in geometry are "'trifles, light as air,' compared with the bewildering question 'What is a Thing?"' So, what did he think a Thing is and how did he come to change his mind?

Carroll had noted in the prefaces to these later editions that he had adopted an alternative definition of *Classification*. He wrote that "this enabled me to regard the whole Universe as a 'Class', and thus dispense with the very awkward phrase 'a Set of Things'." This matter is taken up in Book I, Chapter II. Classification is said to be a process of imagining selecting a certain "Set of Things, all that have a certain Attribute (or Set of Attributes)" (first edition), "Things" (later editions), the result of which is a **Class**. It's unclear why he thought the phrase 'a Set of Things' was awkward. After all, in later editions he made use of the phrase 'the Class of "Things"', which seems more awkward. Nonetheless, Things are prominent in accounting for Classification and *Classes*. And they are the first thing Carroll gets to at the very beginning of Book I, Chapter I, Introductory, where he get right to the point: "The Universe contains Things." Well, what is a Thing? Carroll came to the view that classification is a mental process that can be performed independently of whether the Things being classified exist (are real) or not (are imaginary). This notion had a profound effect on how Carroll treated the problem of existential import in formal logic. In particular, it meant that universally quantified statements of the form 'Every S is P', 'All A are B', 'No P are Q', etc., if they are to be taken as referring to existing things, must be accompanied by an indication of this (e.g., 'Every (existing/real) S is P' or 'Every S is P and something is S'.

It must also be noted here that Carroll, along with his contemporary algebraic logicians, took seriously the idea that statements are always made relative to what was usually called a *universe of discourse*. A universe of discourse is a totality of things and is determined by the context of the statement itself. For Carroll, the universe of discourse could be his "Universe of Things" consisting of both things that exist and things that do not. However, a statement could be made in a context that implicitly assumes a universe of discourse consisting of just existing things, or existing red things, or red planets, or unreal things, or things found only in Wonderland.

3 "A Name of That Thing"

In Book I, Chapter IV, Names, Carroll considered *names* as singular terms or expressions such as proper names ('Plato') as well as what are now called definite descriptions ('the teacher of Aristotle'). As expected, real names "represent" existing things; unreal names represent things that do not exist. In Chapter II he had said that a single thing is a class consisting of just that one thing, an **Individual.** When he came to discussing propositions (Book II), Carroll said that a singular proposition, a proposition with an individual subject, must be taken as a universal statement, one with an implicit universal quantifier (e.g., 'every', 'all'): "A Proposition, whose Subject is an Individual, is to be regarded as Universal." [3, 68] As it happens, this was a policy that had been widely accepted by medieval logicians. Carroll's example was 'John is not well'. His original claim about such a proposition was that it should be properly construed as 'All Johns are men who are not well'. Here, the subject 'Johns' represents the class of men named 'John'. However, he soon came to change his mind [1]. So, in the next two editions, he took the subject term 'John' to represent only the individual (i.e., the one-member class consisting of) John. Finally, in the fourth edition, Carroll sought to clarify his understanding of such singular propositions, by writing that 'John' in his example represents the class of men referred to by the speaker when using that name. Thus, the proposition now becomes: "All the men, who are referred to by the speaker when he mentions 'John', are not well." [1, 68-69] Carroll had been faced here with a logical difficulty that he couldn't easily deal with. The difficulty was due in large measure to his failure, pointed out by Abeles and Moktefi, draw a distinction between what he called a *class* and what logicians now call a *set*. The latter is understood as an abstract object; it is determined by its members but is not identical with its member(s). By contrast, Carroll's class is nothing more than its constituent(s).

Unfortunately, Carroll's logical difficulties regarding singular subject terms did not end there. It's doubtful that he ever got fully clear about the logical syntax of propositions with such subjects. In particular, he seems to have held the old medieval notion of construing these as universally quantified without considering *why*. One reason why traditional logicians had treated singular propositions as universals was that it helped account for the fact that the subject terms of singulars were like those of universals in being *distributed*. Two centuries before Carroll faced the logical difficulty of determining the appropriate treatment of singulars, Leibniz had formulated a solution, one that went beyond the scholastics' *ad hoc* fix. He did so in a single paragraph:

Some logical difficulties worth solution have occurred to me. How is it that opposition is valid in the case of singular propositions – e.g. 'The Apostle Peter is a soldier' and 'The Apostle Peter is not a soldier' – since elsewhere a universal affirmative and a particular negative are opposed? Should we say that a singular proposition is equivalent to a particular and to a universal proposition? Yes, we should. So I when it is objected that a singular proposition is equivalent to a particular proposition, since the conclusion in the third figure must be particular, and can nevertheless be singular; e.g. 'Every writer is a man, some writer is the Apostle Peter, therefore the Apostle Peter is a man'. I reply that here also the conclusion is really particular, and it is as if we had drawn the conclusion 'Some Apostle Peter is a man'. For 'some Apostle Peter' and 'every Apostle Peter' coincide, since the term is singular. [12, 115]

Now, Leibniz accepted, as did all traditional logicians, including Carroll, that singular propositions must be taken as having at least some implicit quantity. What Leibniz meant when he wrote that 'some Apostle Peter' and 'every Apostle Peter' coincide was that, given the presupposition that there is just one Apostle Peter, reference to Apostle Peter was indifferent as to which quantity that might be – because, there being only one such referent, one Carrolian Individual, reference to every Apostle Peter amounts to a reference to at least one Apostle (i.e., to the one, and only, Apostle Peter. Leibniz's solution is semantic, depending on the notion of reference. In such cases as singular propositions, the logical syntax follows. Knowing that some man is a writer, the conclusion that every man is a writer does not immediately follow. However, knowing that (some) Shakespeare is a writer, it follows (not formally, but materially, via the understanding that there is but one Shakespeare) that (every) Shakespeare is a writer. In ordinary discourse, we ignore (indeed, are usually ignorant of) any quantifier here because it makes no difference. As Leibniz said, the two expressions *coincide*.

Traditional logic is a *term logic* (in contrast with modern mathematical logic, which is generally said to be a first-order *predicate logic*). Nonetheless, there are now newer, modern versions of term-logic. In the case of the most prominent one, Leibniz's idea of taking the implicit quantity of singular propositions to be indifferently particular or universal is known as *Leibniz's wild quantity thesis*. For more on wild quantity see: [17], [18], [19], [20], [21], [22, 15-17], [9], [10], [11, 60-64], [23, 73-76].

4 "A Puzzling Question"

Carroll held, in his chapter on Classification (*Symbolic Logic*), that one could form the class of all Things (i.e., the Universe), or one could a class of those Things in the Universe that have some specified property or attribute ("Adjunct" for Carroll). And, since, as we saw, the process of classification is mental, it can be applied to Things whether they exist or not. So, there are no empty classes, classes that have no constituents. For, a class, by Carroll's lights, is

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simply its constituents (it is not a *set* as modern logicians would see it). Logicians say that this view commits one to the principle of *existential import*, which requires that the subject of any universal proposition must correspond (and refer) to things in the universe of discourse. Consequently, in traditional logic, a universal proposition of the form 'Every A is B' logically entails 'Some A is B' (likewise, 'No A is B' entails 'Some A isn't B'). Modern mathematical logicians allow for empty sets. For example, the set of solar planets that are made entirely of ice cream has no members. It follows that they reject the principle of existential import. And that is an important difference between traditional logic and modern mathematical logic.

But there is more to this story. After writing that "the Universe contains Things", Carroll wrote that "Things have **Attributes**." Well, must a thing have attributes? That's one of the questions that philosophers have often found bewildering. Some say yes; some say no. Carroll rejected any answer. In the first chapter of *The Game of Logic*, he wrote this about the question:

People have asked the question "Can a Thing exist without any Attributes belonging to it?" It is a very puzzling question, and I'm not going to try to answer it: let us turn up our noses, and treat it with contemptuous silence, as if it really wasn't worth noticing.

It is interesting to note here that he immediately turned to the related question of whether an Attribute could exist without any Thing, to which he provided a negative reply – in a way only Lewis Carroll could do: "You never saw 'beautiful' floating about in the air, or littered about on the floor, without any Thing to *be* beautiful, now did you?" Carroll was insightful and correct here. There are no Things without Attributes; no Attributes without Things.

A thing with no attributes is called a *bare particular*. Strictly from the point of view of modern logic, there must be bare particulars. This is how perhaps the most prominent American logician of the Twentieth Century put it"

The pronoun is the tenable linguistic counterpart of the untenable old metaphysical notion of a bare particular. [14, 165]

The variable is the legitimate latter-day embodiment of the incoherent old idea of a bare particular. [15, 25]

Quine might not have liked the old idea of a bare particular (he never said why), but he was happy to accept it in its modern version embodied in the new logic. Traditional logic formulates a proposition such as 'Every unicorn is magical' as a Subject ('every unicorn') referring to all unicorns and a Predicate ('is magical') characterizing the referent(s). Modern logic formulates that proposition as 'Everything, (in the domain of discourse) is such that, if it is a unicorn, then it is magical' (symbolically: $(\forall x)(Ux \supset Mx)$). Notice the occurrences of the pronoun 'it' here, which is, as Quine held, the vernacular version of the logicians' *individual variable* (those tokens of 'x' in the symbolism). So, this can, perhaps more transparently be paraphrased as: 'In the domain of discourse, everything (i.e., every bare particular) has the attribute/property of being magical if it has the property of being a unicorn. 'Some lions are tame' would be paraphrased as: 'In the domain, there is at least one bare particular that has the property of being both a lion and tame'. Remember, those bare particulars, now disguised by pronouns/variables, are *bare* (at least until they are accorded properties, such as being a unicorn or being magical, in due course).

5 "The Actual Facts of Life"

Traditional logicians adhered to the principle (called *subalternation*) that a particular proposition (e.g., 'Some logicians are poets' is immediately entailed by its corresponding universal ('All logicians are poets'). As well, they generally held that a particular proposition implicitly entails that something (in the universe of discourse) is referred to by the subject ('There is a logician'). Consequently, the universal proposition entails the existence of at least one thing to which its subject refers. That is the principle of existential import. Modern logicians reject that principle (and thus they reject subalternation). The key difference revolves around how one is to analyze the logic of universal propositions.

Carroll claimed that a universal proposition "contains" its corresponding particular. He gave (*Symbolic Logic*, Part I, Book II, Chapter III) this example:

[Thus, the Proposition "All bankers are rich men" evidently contains the smaller Proposition "Some bankers are rich men."]

And this was a position he held for a long time. Eventually, however, he seemed to have doubts. In Part II of *Symbolic Logic* (Book X, Chapter II), he addressed the issue directly and at some length. There he still held that A propositions contain their corresponding E propositions. But, he also suggested that any theory about this, when "applied to the actual facts of life," must satisfy the test that they not be "singularly inconvenient for ordinary folk." [3, 234]. Even in Part I, Carroll hinted that a different approach to the problem might be taken In Part II: "Note that the rules, here laid down, are arbitrary, and only apply to Part I of my *Symbolic Logic.*" [3, 76] So, given that he came to believe that a logical thesis should not be inconvenient for ordinary folks

when it is applied to "the actual facts of life," what did he come to say about existential import in Part II?

Carroll wrote there that a proposition *asserts* just in case it asserts the existence of its subject. I propositions obviously assert in this way, and so do A propositions, because they *contain* the corresponding I proposition. He then went on to argue that E propositions do not assert, because the assumption that they do leads to a contradiction. Finally, Carroll claims that, one could argue that both A and E propositions assert because any A proposition is logically equivalent to an E proposition (i.e., 'Every X is Y' is equivalent to 'No X is not-Y'). However, that argument would not pass his crucial test (it would be inconvenient and incompatible with the facts of life). The upshot of these considerations is that Carroll believed that A propositions do assert but that the choice then is between taking E propositions to assert or I propositions to assert. He chose to adopt the second choice because of the *inconvenience*, etc. of the first choice. Yet he said that his choice was "the one adopted in this book." It was a policy decision, not necessitated by logic alone. In Note B, at the end of Book X, he appears to suggest that considerations of common linguistic usage are often useful in making such choices. It seems fairly clear that Carroll remained loath to give up the idea of existential import for universal propositions.

6 "Shut Their Eyes Like Frightened Children"

Aristotle was careful to spell out the notion of *opposition*, especially when it comes to terms and propositions (especially in Book X of his *Categorie*). In the case of terms, some pairs are opposite in the sense that one or the other must be true of any subject to which they would apply. For example, 'sighted' and 'blind' are such that for any animal (i.e., a thing to which such terms would naturally apply) just one truly holds. By contrast, a rose cannot be said to be either sighted or blind. For Aristotle, a term like 'blind' was a "privative" term. Pairs consisting of a term and its privative admit no intermediaries. Whatever is, by nature, one cannot be the other. They are (logically) *contraries*. However, there are many pairs of terms that cannot both the true of the same thing and the same time, but they have intermediaries that can hold along with one or the other. For example, 'good' and 'bad' are contrary (in the sense that they cannot both simultaneously hold of the same subject, Nevertheless, there are people who are not bad but nonetheless are not necessarily good. They're somewhere in between. Such pairs of terms are contraries – but not logical contraries. In his On Interpretation (Chapter II) [2], Aristotle wrote that terms that are privative are "indefinite" terms of the form 'not-x'. So, terms seem to come in pairs, 'x'/ 'not-x'. But so do propositions.

Two propositions opposed in this way are *contradictories*. Aristotles's account here became a part of the very foundation of traditional logic.

Carroll, unsurprisingly, accepted this tradition of distinguishing between the negation of a term and the negation of a proposition. Yet many of his contemporary logicians (and ultimately, most of subsequent mathematical logicians) did not. Referring to the former group, he wrote:

The fact is, "The Logicians" have somehow acquired a perfectly *morbid* dread of negative Attributes, which makes them shut their eyes, like frightened children, when they come across such terrible Propositions as "All not-x are y" and thus they exclude from their system many very useful forms of Syllogisms. [3, 238]

He had written something quite similar at the end of Chapter I of his *The Game of Logic*.

Throughout his works on logic, Carroll was happy to use negative terms, which are, after all, quite common in English. His confidence in recognizing the role of term negation for logic was born of his clear-eyed understanding of the difference between contradictory propositions and negative terms. Early on in *Symbolic Logic* (Book I, Chapter III) he wrote this about the latter:

Henceforwards let it be understood that, if a Class of Things be divided into two Classes, whose Differentiae have contrary meanings, each Differentia is to be regarded as equivalent to the other with the word "not" prefixed.

In other words, *logically contrary* terms, which he confusingly calls *contradictory* terms (Bartley 285-286), divide a class exclusively and exhaustively – just as Aristotle had taught. In the next chapter, Carroll admits negative terms even when they are in the subject, the very thing that most frightened "The Logicians," showing that, for example, 'None but the brave deserve the fair' is equivalent to 'No not-brave deserve the fair'. He did not go on to recognize that 'no' itself is not, as he thought, a quantifier. It is a portmanteau word, analyzable as 'Not: some/any'.

Modern logicians understand the contradictory of a proposition as the result of applying function to the proposition, a function of *propositional negation*. Carroll, good traditionalist that he was, understood the contradictory of a proposition to be its *denial*. And the denial of a proposition is the result of two things: change of quantity and change of quality. The first change involves making a universal a particular or a particular a universal (i.e., an exchange between A and I or an exchange between E and O). The second change involves exchanging a positive copula for a negative copula. Importantly, a negative copula is not a negative term.

7 "With Bated Breath"

All Carroll tells us about logical a copula in Book II, Chapter I, of Symbolic Logic is that it is "The verb 'are' (or 'is'). (This is called the **Copula**.)" Not much more light is shed on the topic when he mentions it again in Symbolic Logic, Part II, Book X, Chapter II. Yet it is still worth quoting in full. After all, only Carroll could mention the topic in such a delightful way.

The writers, and editors, of the Logical text-books which run in the ordinary grooves - to whom I shall hereafter refer by the (I hope inoffensive title "The Logicians" - take on this subject, what seems to me to be a more humble position than is at all necessary. They speak of the Copula of a Proposition "with bated breath," almost as if it were a living, conscious Entity, capable of declaring for itself what it chose to mean, and that we, poor human creatures, had nothing to do but to ascertain *what* was its sovereign will and pleasure, and submit to it.

He took a *slightly* more serious tone in the following chapter, where he directly addressed the question of whether one should, when appropriate, attach a 'not' to the copula or to the predicate. In other words, what, if any, is the logical difference between propositions of the forms 'Some S is-not P' and 'Some S is not-P' (e.g., 'Some head of state isn't competent' and 'Some head of state is incompetent')? His answer:

When they [The Logicians] are putting the final touches to the grouping of their Propositions, just before the curtain goes up and when the Copula – always a rather fussy "heavy father," asks them "Am I to have the 'not,' or will you tack it on to the Predicate?" they are much too ready to answer, like the subtle cab-driver, "Leave it to you, Sir!" The result seems to be, that the grasping Copula constantly gets a "not" that had better have been merged in the Predicate, and that Propositions are differentiated which had better have been recognised as precisely similar.

Carroll's choice to treat the two alternative propositional forms as logically equivalent amounts to the traditional rule of immediate inference called *obversion*. In effect, one use of the rule is to eliminate negative copulae in favor of a negative predicate term. As it happens, a system of formal logic that systematically admits both negative copulae and negative terms has important advantages inference (see [16, final paragraph]). But that's a story for another day.

A brief final note: The work presented here is the direct result of a suggestion, followed by much encouragement, then followed by invaluable input from a first-class historian of Nineteenth Century British logic and Carroll's place there. Without all of that, this essay would not be a Thing existing in the Real, rather than Imaginary, Universe. I thank and dedicate this essay to Amirouch Moktefi.

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