South American Journal of Logic
Vol. 5, n. 2, pp. 179–195, 2019
ISSN: 2446-6719

Essay on Perspectivism in the Philosophy of Science

Décio Krause

Abstract

We articulate a notion of perspectivism in the philosophy of science based on the following grounds: (1) there are different and in general non-equivalente ways of approaching a certain domain D of knowledge, and we cannot justify the preference by one of them except by appealing to pragmatic criteria; (2) each of these different approaches originate distinct informal theories about the domain, and each of them may give rise to diverse axiomatic or formal theories; (3) these perspectives capture aspects of the domain, although no one of them can be, stricto sensu, true about D; (4) it is necessary to pay attention to the metamathematics we use for both to formulate the theory and to discuss its models. Thus, our perspectivism does not reduce to relativism, and it seems to be in accordance with the present day philosophy of science. The plurality of perspectives (logical, mathematical, metaphysical, and so on) should be viewed with care by the philosopher, who should note that the concepts and assumptions she usually makes is context dependent (we mean, metaphysically, logically, and mathematically dependent).

We don't try to put our view as a definitive one, nor try to convince anyone, but just suggest to the reader to cinsuder the remarks of this text as a possibility; as Ortega said in his *Meditaciones del Quijote*, "Yo sólo ofrezco — modi res considerandi —, posibles maneras nuevas de mirar las cosas. Invito al lector a que las ensaye por sí mismo, que experimente si, en efecto, proporcionan visiones fecundas: él, pues, en virtud de su íntima y leal experiencia, probará su verdad o su error."

Keywords: Perspectivism, pluralism, scientific theories, axiomatization, logic.

> "La verdad, lo real, el universo, la vida — como queráis llamarlo — se quiebra en facetas innumerables, en vertientes sin cuento, cada una de las cuales da hacia un individuo. Si éste ha sabido ser fiel a su punto de vista, si ha resistido a la eterna seducción de cambiar su retina por otra imaginaria, lo que ve será un aspecto real del mundo. Y viceversa: cada hombre tiene una misión de verdad. Donde est mi pupila no está otra; lo que de la realidad ve mi pupila no lo ve otra. Somos insustituíbles, somos necesarios (...). Dentro de la humanidad cada raza, dentro de cada raza cada individuo es un órgano de percepción distinto de todos los demás y como un tentáculo que llega a trozos de universo para los otros inasequibles. La realidad, pues, se ofrece en perspectivas individuales."
>
> José Ortega y Gasset, *Verdad y perspectiva*, [21, p.19]

Introduction

I am in glad with the invitation to submit a paper to this volume celebrating the 10th anniversary of our Logic Working Group (Grupo de Trabalho em Lógica, GT-Lógica) of the Brazilian Association for Post-Graduate Studies in Philosophy (ANPOF). During the years, I have made some communications to this GT in different subjects, but most of them centred in a huge problem I have been working in, namely the logical and metaphysical (or ontological) aspects of quantum theories, in special the problems regarding the non-individuality of quantum objects and its discussion.

But, in this contribution, I will be more general and speak of a particular view about some topics which I think have been neglected by philosophers. I don't know the reasons for this actitude; perhaps they don't see some things from the perspective I do, or simply because they don't think that my questionings are relevant. Anyway, I shall leave the final answer to the reader. As Ortega says, "Yo sólo ofrezco *modi res considerandi*, posibles maneras nuevas de mirar las cosas" [22, p.25] ("I just offer *modi res considerando*, possible new ways of looking to the things"). Furthermore, this is done in a form of an essay, an *intelectual love*, as Ortega says paraphrasing Spinoza [22, p.8].

We consider how some scientific theories could had grown[1] from the supposition about the existence of a world that can be investigated to the ways we standardly (some exceptions shall me mentioned in between the text) work to elaborate our formal theories about parcels of such a reality. I do not claim that the scheme presented below captures *any* scientific theory but, as we shall see, it maps a general strategy almost common and enables us to identify some

[1] In the sense of a Lakatosian "rational reconstruction" [16].

relevant points in the discussions about the foundations of scientific theories.[2]

My point is to start by acknowledging that there are different perspectives (as I prefer to call them) both to interpret the raw stuff of data we get in the laboratory, and also to consider the way we elaborate the theories, initially (in general) from an intuitive point of view and them (in some relevant cases) from an axiomatic or even from a formal point of view. There are different perspectives also from the metaphysical side, for there is no way to ensure that just one conception about the world is shared by philosophers and scientists. The necessity of calling the attention to these points seem almost trivial to some, but as we shall see, they involve unsuspected consequences a philosopher should also consider and explore.

Perspectivism, as I see it, is the thesis that a same object,[3] in particular a physical object, a text, a human society, a domain of knowledge, whatever, can be viewed from different perspectives, and theoretically elaborated from different perspectives. There is no reason for supposing that evolution and genetics force us, the theory-builders, reason and theorize in just one necessary way. In other words, I strongly suspect that it is false to suppose that due to te way we have evolved, *necessariy* we would arrive to what we usually call *classical logic* and to other "standard" ways of reasoning. These theories and ways of reasoning are contingent. Other ways of getting a *Weltanshauung* than that one which is particularly ours are logically possible. Logic is not *a priori*; there are several alternative and non compatible systems of logic so as there are several alternative and (in the whole) incompatible geometries. Paul Cohen, in the 1960s, coined the term *non-Cantorian set theories* to characterize those set theories that depart from the "standard" one; and, of course, their corresponding mathematics could also be termed *non-Cantorian mathematics*.[4]

But we could say that we have different *theories*, but just one *science*.

---

[2]For instance, intuitionistic mathematics surely cannot be put within such a cake pan.

[3]The word 'object' is used here in the sense of Carnap; as he says, "[t]he word "object" is here always used in its widest sense, namely, for anything about which a statement can be made. Thus, among objects we count not only things, but also properties and classes, relations in extension and intension, states and events, what is actual as well as what is not." [2, p.5]

[4]As Cohen and Hersh say, "The analogous development with respect to non-Euclidean geometry — what we might call non-Cantorian set theory — has taken place only since 1963, in the work of one of the authors of this article (Cohen). What is meant by 'non-Cantorian set theory'? Just as Euclidean and non-Euclidean geometry use the same axioms, with the one exception of the parallel postulate, so standard ('Cantorian') and nonstandard ('non-Cantorian') set theory differ only in one axiom. Non Cantorian set theory takes the axioms of restricted set theory and adds not the axiom of choice but rather one or another form of the negation of the axiom of choice. In particular we can take as an axiom the negation of the continuum hypothesis. Thus, as we shall explain, there now exists a complete solution of the continuum problem for characterizing those mathematics that can be developed within a mathematical framework that denies the Continuum Hypothesis."

However, even this hypothesis can be questioned. The demarcation between what is scientific and what is not (the so called *the demarcation problem*) is not clearly answered, despite some like Popper claimed to have solved (see the chapter "The Problem of Demarcation" in [24]) the problem but not without suspicions, such as those of Paul Feyerabend [28]. Science can also be seen from different perspectives: social, economical, logical, methodological, metaphysical, epistemological, etc. All perspectives we use for considering the scientific stuff contribute to the whole vision of the subject; as Ortega said in the last sentence of the above motto, "[r]eality, therefore, offers herself in individual perspectives". Let us explore this idea a little in what follows.

1 The general schema

Suppose that we have a domain of the empirical sciences to investigate, say species transformations, or evolution (it could be also a domain of pure mathematics, say number theory — the adaptations to these cases are obvious); let us call it "D". Of course in such an investigation we may be interested in various aspects (physical, chemical, biological, social etc.) of the objects of that domain, the animals and plants, so as in many relationships we assume there are among them, which may be important for the development of a theory of evolution. At first glance, we theorize from experimental or phenomenological data, using previous knowledge we bring with ourselves and which we believe deserve some credibility. We use deduction and induction (in general, both informally), although sometimes with great level of rigor, at least in the case of deductions, which are generally made with the help of some mathematics.[5] We also make use of other previous theories, cultural background, which carry influence on our way of looking to D, such as inspiration, and insight. There are no limits for imagination, and all we can hope is that our final theory agrees with observation and experience, and enables prediction. Apparently, that is all. History of science has shown different ways of noting the details of such an endeavour, with (as is well known) distinct opinions and descriptions by philosophers and historians of science.

Depending on several factors (which include historical, cultural, ethical, etc.), our investigation of the domain D may conduce to several different and non-equivalent informal theories about the domain, which can be seen as providing different perspectives about that parcel of reality. For instance, it is sometimes taken form grant that the Western and the Eastern ways of looking to the world differ substantially, but it is not of this kind of differences we

[5] Georg Kreisel coined the term "informal rigour" to refer to *rigorous ways* of mathematical reasonings can be done even at an informal level when "doubtful properties and intuitive notions" are eliminated [15].

are talking about. We are committed with the different ways of formulating a scientific theory about a certain domain; Heinsenberg's and Schrödinger's accounts to quantum mechanics are typical examples of different approaches, although their theories are in a certain sense equivalent — more on this below. But sometimes we get informal approaches which completely do not agree with one another, sometimes being in direct confrontation, mainly if the field under investigation is still something substantially new. For instance, present day physics, trying to link quantum physics and gravitation, seems to run on two parallel but quite distinct and non equivalent roads: string theories and quantum gravitation in loop, yet a final mixture of both cannot be discharged (for general readings, see [9] and [26]). Some of the formulations we achieve of course may be proved to be wrong, as Lysenkoism was (we mean the genetic theory by Trofim Lysenko, the Russian biologist of Stalin's period). Strictly speaking, our investigations always presuppose something: there is no tabula rasa investigation, free from some theoretical context. As Heisenberg remembers, he learnt from Einstein that "It is the theory which decides what can be observed" [11, p.10]. The same can be said concerning our scientific formulations: we always investigate linked to theoretical credos, although we sometimes can (and should) leave them.

The growing of scientific knowledge is a subject that has been investigated from ancient times, and if we leave the interest in the domain properly and pay attention to the meta-theoretical analysis were the theories are build, we are faced with lots of philosophical questions, mainly if we take into account the recent (that is, from the 20th century) developments in the fields of science, mathematics, and logic. Really, the implications of non-classical logics, non-Cantorian mathematics, various conceptions of probability, different conceptions of truth, and so on, present to the philosopher a plurality of perspectives that deserve careful discussion and explanation.

Analyzing the domain D, our scientific activity suggests at first glance an informal theory, formulated in the natural language, eventually supplemented by additional symbols and mathematical concepts, so as concepts from other sciences, like physics. Darwin's natural selection theory can perhaps be taken as an example of such an informal (not axiomatized) theory. In a certain sense, an informal theory is something similar to a Carnapian framework [1]. Sometimes such an informal theoretical formulation is termed prototheory, pretheory or even mathematical model, but we shall use the term informal theory here, for we deserve the words "theory" (tout court) and "model" to be used later. Really, as we said, we may arrive at more than one informal theory, depending on the scientist's backgrounds and experience. For instance, concerning evolution, we know that Darwin's natural selection theory provides an example of an informal (not axiomatized or formalized) theory, so as are sexual selection and pangenesis (the Darwinian theory of heredity). In a certain sense, these different perspectives are so that some of them may be abandoned later (as pangenesis was), but we may say that all of them contribute to a better understanding of the domain, which by its peculiar richness (mainly in the domain of the empirical sciences), cannot be captured by a single approach. In fact, probably Darwin himself believed that all three above mentioned processes are essential to a better understanding of evolution.

Thus, the first point to emphasize is that, given a domain D, there may be (in principle) several distinct informal theories about D (or i-theories, the 'i' standing for 'informal') which do not need to be compatible one each other. But, being informal theories *about* D, there are certain key concepts and assumptions that induce the belief that all of them are about the same domain of knowledge. In our example, we can mention key concepts such as selection, species, and several others. If D is taken to be microphysics, we can mention the informal theories of Heisenberg (matrix mechanics), of Schrödinger (wave mechanics), Bohm's theory, Dirac's theory for the electron and so on. In general, all of them are formulated within informal mathematics and proceed informally, that is, not axiomatically.[6]In this last case, despite their differences, there are also some key concepts that identify them as "quantum mechanics", such as the uncertainty principle, superposition, entanglement, the strict dependence on probability, postulates of symmetry, incompatibility of certain observables, and of course the use of Planck's constant, yet in some cases the concepts need further clarification.

2 From informal to formal theories

Usually, the scientist like a biologist becomes satisfied with the informal theories she has and which has proven its success, for it became useful for explanations and predictions. The "conclusions" she gets are then re-interpreted in terms of D and then she believes that her theory is accurate and good if the results are in accordance with the experiments or, as some say, if it "saves the phenomena" [31, Chap.3]. But the philosopher is usually not satisfied with such a schema, and asks for a step further. Let us say something about this next step.

Given an informal theory IT (we insist that there may be several of them), we can continue the theoretical inquiry about IT by using the axiomatic method, which is perhaps the best methodology we have for keeping a theory sufficiently precise in regarding its foundational aspects.[7] This meta-theoretical analysis

[6]Even in mathematics this is usually so. Differential and Integral Calculus is not tough my means of axioms, so as Analytic Geometry, although t hey could.

[7]We should not discard the precision that can be achieved even if the theory is not ax-

may originate basically two kinds of theories: an axiomatic theory, obtained as usual by making explicit its primitive concepts and postulates (according to one possible approach), but without detailed explanation, but presupposing, the underlying logic, which may encompass a set theory. Or then one can go to a formal version of IT, which provides explicitly all the details concerning the theory, starting with its basic language, formation rules, basic logic, and so on. The choice between these procedures depends on the level of rigor we are interested in. In a certain sense, the level of rigor we get is associated to the emphasis we give to the axiomatic method.

Generally speaking, we have basically two approaches to treat a certain theory axiomatically. The first is called internal method, and is performed within a certain mathematical stuff, say the Zermelo-Fraenkel set theory (which can be assumed here without lost of generality). Assuming ZF and its underlying logic (say, first-order classical quantificational logic with identity), all we need is to provide the theory's specific axioms, written in the language of ZF perhaps extended by additional specific symbols. This is basically Suppes' approach, where the axiomatic version of the informal theory is done by means of a set theoretical predicate [30]. In this kind of formulation, despite the abstractness of the approach, there is always an intend content of the basic concepts: 'selection' has a precise sense in biology, and superposition has also (albeit a disputed one) a meaning in quantum physics. The meaning of these concepts is (at least in principle) captured by the informal theories of D. Thus, the models of the theory are set-theoretical structures built *within* ZF, that is, they are *sets* in ZF that satisfy the predicate. Of course, this approach cannot be used to define, say, ZF proper, for we would be supposing the existence of a model of ZF within ZF, which is impossible due to Gödel's second incompleteness theorem (being ZF consistent).

Before to continue, a remark that shows the relevance of the mathematical meta-framework. One of the alternatives for the foundations of mathematics, and which has found applications in physics, computacional sciences and in many other fieds, is category theory [18]. The notion of category can be given from different ways, including a first-order formulation [10]. Let us think of this case. We have a first-order theory T whose models are the categories. But, *where* these models are to be build? They cannot be constructed *inside* a set theory like ZF, for these models are only what mathematicians call *small categories*. In order to achieve the "right" theory, it is necessary to consider also "big" categories such as Set (the category of all sets), Top (all topological spaces), AbelGroups (all Abelian groups), VectSpaces (all vector spaces), Cat (all small categories), and so on, which cannot be seen as sets in ZF.[8] That is,

iomatized. In [14], we discuss with more details the process of axiomatization and its role.

[8]The general reader does not necessitate to take into account what are these categories

we need to discuss also where these models are to be found, and realize that they in a certain sense will depend on the characteristics of this framework (more on this below).

Of course that other logical basis are also available. For instance, much of empirical science could be done within a higher-order logic, despite the difficulties that surely would appear,[9] or in category theory (for a clear development of parts of physics using categories, see [7]; for higher-order logics, see [3]). But here we shall keep with set theories.

The other approach is called external, and in this case we define all the mathematical details concerning the theory's stuff. Thus, we may say that, at a first glance, a theory encompasses three levels of postulates: (i) the logical postulates, say first-order classical logic; (ii) the mathematical postulates, say ZF set theory, and (iii) the specific postulates of the theory, say group postulates, field postulates, classical particle mechanics postulates, and so on. It may be also addressed that this scheme is quite general; we could suppose that as the background theory we are using Morse's set theory instead of ZF (this applies also for the internal approach), which strictly speaking has no "underlying logic",[10] thus item (i) could be dispensed with. As we have said, we could also use higher order logic for covering steps (i) and (ii) or category theory instead. This schema is quite general, but serves for our purposes. Here we shall suppose that T is the axiomatic version of the informal theory (IT); as usual, we shall assume that if we use a set theory like ZF, we are implicitly supposing a certain underlying logic. The other possibility is the formalization of IT, by depriving IT from any meaning, which will be acquired only when the formalism is interpreted. Generally, this is achieved by prescribing all the basic stuff concerning IT, starting from a basic language, formation rules, logical postulates and so on.

Anyway, we can suppose that once axiomatized, a theory keeps abstract, a kind of Popper's third world entity [25, Chap.3], getting the possibility of different interpretations than that one which originated the theory, which we can call the intended interpretation.

The next step is to be aware that, given a certain informal theory IT, there are also several possible theories (formal of axiomatized) T associated to a same IT. A particular one will depend on the particular primitive concepts we choose and the way we conduce the axiomatization. We shall suppose that an

specifically. It is enough to acknowledge that they are "big" enough to fit inside a set theory like ZF.

[9] For instance, higher-order logic does not express the so-called Cantor's transfinite mathematics. Furthermore, it is more difficult to deal with, mathematically speaking.

[10] As is known, in Morse's approach, set theory is developed directly, without commitment with an "underlying logic"; see [20].

exchange of language and/or postulates conduce to a distinct theory, yet this can be disputed. Thus, the different axiomatizations of classical propositional calculus, although conducing to the same logical truths, are considered here as distinct calculi, all of them in a certain sense equivalent. But we should be aware that all these formulations are given in a certain mathematical background, the metatheory we use to speak about our theory T and for formulating its rules and principles. The surprising fact is that, usually, philosophers do not take into account the fact that we can choose, say, different set theories to axiomatize the informal theory. What are the consequences of such a choice? As we shall see, there are important consequences, which bring the question of justifying a particular choice. But before discussion this specific topic, let us continue a little bit with our general scheme.

3 The importance of the metamathematics

Let us consider now a particular theory T. An example in evolution is M. B. Williams' formulation of Darwin's theory of evolution [32]; in microphysics, we may think of Mackey's well known axioms for quantum mechanics [17], Wightman's axioms for quantum field theory [27], or then Einstein's field equations for general relativity [23, p.462]. Suppose that T is axiomatized (or adequately formalized) by a set-theoretical predicate in Suppes' sense [30]. The predicate defines a *species of structures*, and the structures that satisfy the predicate are the structures of that species, or the models of the predicate. For instance, the predicate "x is a group" defines a species of structures, and the groups are the structures of this species.

In the case of empirical science, we usually regard D as the intended model of T, but this assertion must be qualified. In fact, a vague domain of experience cannot be a model of a certain mathematical theory in a precise sense. Really, we do not work with D directly, but with a mathematical representation of D, which originates a certain mathematical structure that is to be one of the models of the set-theoretical predicate. Thus, only indirectly we can regard T as referring to D. Furthermore, and this is what we are claiming to call your attention to, the metamathematical stuff we chose to formulate such a "model" may vary. Generally speaking, we can take the informal theory IT to play the role of this first mathematical formulation of the theory of the domain — applied scientists say that they have "modelled" T. The theory T properly refers to D only indirectly, via IT. The link between T and IT (adequately formulated) can be seen from the point of view of standard semantics, yet there are problems concerning higher-order theories, for as it is well known, we don't have an adequate model theory. We shall return to this point below.

Let us summarize what we have up to here: we began with a wide and generally vague domain of knowledge, D, and have verified that we can formulate various informal theories IT about D. Each informal theory, by its own, may originate various axiomatic or even formal theories T, which only indirectly refer to D. Now let us consider a particular theory T. As an abstract mathematical theory, yet physically motivated, T (supposed consistent) may have several different *models* in the sense of standard model theory (perhaps T is categorical, but this is another point we can leave outside this discussion for a moment). These mathematical models are mathematical structures that verify the postulates of T and can reflect the possible domains of application of the theory. Sometimes the theory T is confounded with the class of its models, as the philosophers who defend the semantic view of theories usually claim [31].[11]

The empirical domain D can be said to be the intended (or intensional) "model" of T. But it is a "model" only in an informal point of view (there are two senses of the word "model" being used here; the first concerns the structures that *model* the postulates (or te set-theoretical predicate, which is the same), while the second sense is informal, meaning the domain we suppose the theory applies to. Of course T refers to D only indirectly, for we can't associate empirical content to mathematical concepts directly. We do it but only via a mathematical model of D we formulate using some (in general, very high) mathematics. For instance, take the models of general relativity. It is today well known that there are several non-equivalent solutions of Einstein's field equations. All of them can be taken as possible realities, including Gd̈el's model with closed time lines, which enables travels to the past [8].

The variety of models of a theory T, once formulated as an abstract mathematical theory, raise, as we see, lots of interesting philosophical questions. But there is more. The variety of possibilities we have in choosing the metamathematical framework where we formulate the theory, also present us interesting philosophical questions. First of all, we need to see that such a variety of possibilities (a form of pluralism) offer us various perspectives about the domain D. Each one of them reflect aspects of D and can be seem as a partial view about the domain. No one of them captures D in totum, and the choice among them can be made only by pragmatic criteria, like intuitiveness, simplicity, or expressive power. There is no relativism here. We refuse the view that any choice can be taken with equal value, but the particular choices of these frameworks are theoretically dependent, and we can justify a particular choice by appealing even to metaphysical criteria, although we prefer to justify the choice by means of pragmatic criteria.

Questions like these ones cannot he answered from the inside of the particular theory (axiomatic or intuitive). They are more or less like Carnap's

[11] This kind of talk is of course vague, for a model is a model of something and classes of models do not exit per se, but need to be formed as models of a system of postulates.

external questions [1] but, contrary to him, we repute them as extremely important. For instance, we can ask: where the theories T and their models are formulated? This raises the question of the metamathematical framework we use to formulate T (and its models), in the same sense we have already mentioned with respect to category theory. Suppose it is ZFC, the standard Zermelo-Fraenkel set theory with the Axiom of Choice (AC) in one of its usual presentations [5]. As it is well known, due to AC, we can prove that there are sets of real numbers that are not Lebesgue measurable.[12] But we also know ever since Gödel (1938) and Cohen (1963) that AC is independent of the remaining axioms of ZF (supposed consistent). Thus there is Solovay's model of set theory (ZF), which uses a form of negation of AC, and it results that in such a model every set of real numbers is Lebesgue measurable. These are apparently only mathematical differences, but we must recall that the same was said about those paradoxes of set theory, like Burali-Forti's or Cantor's, they were also once considered so "distant" from day-to-day mathematics. that did not deserve much attention. Furthermore, there are other differences between standard ZFC and non-Cantorian set theories we have seen above. The philosophical investigation of these frameworks, so as of the theories built within these frameworks, is still open for discussion, so is analysis like what kind of "physics" could we obtain using Solovay's model.

Another example in which a different (to standard informal mathematics) metamathematics can be used may be the following: in Quine's set theory NF (New Foundations), AC is false [6]. Hence, in the corresponding mathematics, which can be built in NF, we can't obtain those results which depend essentially from AC. If NF (or other) is taken to be our methamatical framework, which are the consequences? Of course this needs to be explored also in connection to empirical sciences, for the mathematics we use in this field generally presupposes AC. Another example: In NF, mathematical induction holds only for "stratified" formulas, but not in general. But usually we use induction for in several places in our philosophical investigation and discourse; for instance, in defining formulas in elementary logic. In Tarski's concept of truth, we also use induction to define when a formula is true in a certain structure. We guess that all these formulations can be done with stratified formulas, but we are not sure concerning all uses of induction we make in science. So, maybe the informal use we do can be not in agreement with the strict metamathematics we are working with (NF, say). This plurality of possibilities is a question to be explored by philosophers.

Thus, form the above remarks we should acknowledge that the models of a certain scientific theory (either informal or axiomatized/formalized) are

[12]This (and other) kinds of "measure" generalize the standard way we say, for instance, that the distiance between points a and b in the real line is given by $|a-b|$.

usually built in a certain metamathematics, which we can assume to be a set theory. For the sake of precision and without loss of generality, we can assume that such a set theory is ZF. In this case, "classical particle mechanics" (that is, the models or systems of a classical particle mechanics) emerges from structures of the form $\langle P, \vec{s}, m, \vec{f}, \vec{g}\rangle$, where P is the set of "particles", $\vec{s}$ is the position function (vector), m is the mass function, $\vec{f}$ stands for the sum of the internal forces, and $\vec{g}$ represents the external forces, all of them obeying certain postulates, put by McKinsey, Sugar and Suppes in 1953 [30, pp.319ff]. As for non-relativistic quantum mechanics, a mathematical structure that can be taken as a model of this theory is something like $\langle M_0, S, Q_0, \ldots, Q_n, \rho\rangle$, where M_0, the mathematical part of the structure, is a model of standard functional analysis, while $\langle S, Q_0, \ldots, Q_n\rangle$ is the "operative part" of the structure, and ρ is an interpretation function that assigns an element of M_0 to each element of the operative part; and, once again, each of these components also obey specific postulates [4, p.85] (an alternative structure can be seen in [13, §5.8.1]). In principle, all physical theories can be described within such a schema (but see below).

As we see, there is sensitivity to language, and there is a (meta)mathematical framework in which these structures are built, and the lack of consideration of this point may bring considerable problems, particularly if we take into account contemporary physics. Let's consider an example. An important concept in quantum mechanics is that of an unbounded operator. For instance, the position and momentum operators in the Hilbert space $\mathcal{L}_2(R)$ of the equivalence classes of square integrable functions, are unbounded. An operator A is unbounded if for any natural number $M > 0$ there exists a vector α such that $\|A(\alpha)\| \geq M\|\alpha\|$. However, consider the theory ZF+DC, where DC stands for a weakened form of the axiom of choice (the axiom of *dependent choices*) entailing that a 'countable' form of the axiom of choice can be obtained. In particular, if $\{B_n : n \in \omega\}$ is a countable collection of nonempty sets, then it follows from DC that there exists a choice function f with domain ω (the set of the natural numbers) such that $f(n) \in B_n$ for each $n \in \omega$. It can then be proven, as Solovay showed, that in ZF+DC (which is supposed to be consistent) the proposition "Every subset of $\mathbb{R}$ [the set of real numbers] is Lebesgue measurable" cannot be disproved. It is not important for the moment to discuss the precise concept of Lebesgue measure, but just to accept that this proposition is false in standard ZFC. The same happens with the proposition: "Each linear operator on a Hilbert space is bounded" [19]. This kind of result poses a difficulty to the defenders of the semantic view: when we speak of the models of a scientific theory, such as quantum mechanics, which metamathematics should we use to define its models? Presumably, it cannot be Solovay's model in ZF+DC, since we need unbounded operators. So, the choice of a suitable metamathematics is crucial.

Here is another example. In the standard Hilbert space formalism, we deal with bases for the relevant Hilbert spaces. More specifically, we deal with orthonormal bases formed by eingenvectors of certain Hermitean operators. This is possible because we can prove, using the axiom of choice (which is part of the metatheory used here) that any Hilbert space $\mathcal{H}$ has a basis. Moreover, it can also be shown that each basis has a specific cardinality, which is the same for all bases of $\mathcal{H}$ (this is defined to be the dimension of the space). But in physics apparently it is more interesting to make use of set theories comprising ur-elements, that is, entities that are not sets but that can be elements of sets to cope with physical objects. Thus, let us suppose that our theory involves ur-elements. Then, in certain set theories with ur-element and in which the axiom of choice does not hold in full generality, such as in the so-called Laüchi's permutation models, we obtain: (a) vector spaces with no basis, and (b) a vector space that has two bases of different cardinalities [12, p.366]. Now, if a vector space has no basis, it cannot be used as part of the standard formalism of quantum mechanics. The latter formalism presupposes the availability of suitable bases. As a result, the formalism depends crucially on the metamathematics that is used.

Despite all the discussion about the concept of 'model' of a physical theory given in the literature, the precise characterization of this concept remains elusive. Model theory, which has been the inspiration for much that has been said on models of scientific theories in general [31], articulates the notion of a model for formal first-order axiomatic systems only. Due to the fact that fundamental theorems, such as compactness, completeness and Löwenheim-Skolem, do not hold in higher-order logics, we can say that there is no higher-order model theory, so, we really don't know in a precise way the theorems we can consider to hold for the models of a scientific theory which in general use more than first-order logic.[13] Despite this, a model for a scientific theory in standard texts on the semantic view is typically taken in its 'first-order' sense, roughly, as a set-theoretic structure encompassing one or more domains and relations having as relata the objects of these domains, and not higher-order relations. But it is easy to realize that, say, classical particle mechanics [30], orthodox quantum mechanics [17] and several other theories are not formulated as having elementary models of this sort (in mathematics, it suffices to remember well-orderings, topological spaces and many other basic structures) [13]. And more, it would be doubtful if they can. Really, suppose we have a "legitimate" first-order axiomatization of classical particle mechanics out of set theory (a Herculean

[13] The fact that set theory can be formalized as a first-order theory needs to be analysed carefously, for the expressive power of the set-theoretical language is stronger than that of elementary languages.

task to be done), perhaps in the sense of tyhe logical positivists [30, 13]. In this theory, we use the set of natural numbers with all its properties, so, they need to be axiomatized as a first-order theory. Thus, this arithmetics shall have models. But, which model of this first-order arithmetics should we when quantify over natural numbers? Could we use a non-standard one? [5, p.299] The same can be applied to real numbers of course. Furthermore, even if we make use of set theory, as it is common, if we need to consider some particular model of first-order set theory, which one we choose? For doing physics, can we chose the denumerable model that exists due to the downward Löwenheim-Skolem theorem? It is clear that all these perspectives need to be considered in foundational analysis.

All these themas are, as the reader surely agrees, originated from the possibility of having diverse perspectives to look to a same subject. This clearly poses a pluralism of possibilities of investigation, and it would be one of the tasks of the philosopher interested in foundations to consider them.

4 Summing up

In this paper, we articulated a pluralism of perspectives in the philosophy of science based on the following characteristics: (1) there are several ways of approaching a certain domain of knowledge, and we cannot justify the preference by one of them except by appealing to pragmatic criteria; (2) these different approaches originate distinct informal theories about the domain, and each of them may give rise to diverse axiomatic or formal theories; (3) each one of these perspectives captures aspects of the domain, and although no one of them can be said to be true stricto sensu, they help us in getting a clearer idea of what have been investigated; (4) there is an evident necessity of considering the framework where we discuss the theories. Thus, our perspectivism does not reduce to relativism, and it is in accordance to the present day developments in science (formal and empirical). Our perpectivism is a way to agree with Ortega's fight against any tentative of keeping the philosophical world in a closed universe, expressed in his *Meditaciones del Quijote* [22]. The plurality of possibilities (logical, mathematical, metaphysical, and so on) should be viewed with care by the philosopher, who should note that the concepts and assumptions she usually makes is context dependent, so, no analysis should be done without first specifying *where* (on in what grounds) it will be performed.

References

[1] R. Carnap. Empiricism, semantics, and ontology. *Revue Internationale de Philosophie* 4, 20-40, 1950.

[2] R. Carnap. *The Logical Structure of the World.* Chicago and La Salle: Open Court, 2nd.priting, 2005.

[3] R. Carnap. *Introduction to Symbolic Logic and Its Applications.* Dover, 1958.

[4] M.L. Dalla Chiara and G. Toraldo di Francia. *Le Teorie Fisiche: Un'Analisi Formale.* Torino: Boringhieri, 1981.

[5] A.A. Fraenkel, Y. Bar-Hillel, Y. and A. Levy. *Foundations of Set Theory.* Amsterdam: North-Holland, 1973.

[6] T. Foster. Quine's New Foundations. *The Stanford Encyclopedia of Philosophy.* (Summer 2006 Edition), Edward N. Zalta (ed.). http://plato.stanford.edu/archives/sum2006/entries/quine-nf/

[7] R. Geroch. *Mathematical Physics.* Chicago and London: The Un. of Chicago Press, 1985.

[8] K. Gödel. An example of a new type of cosmological solutions of Einstein's field equations of gravitation, In Gödel, K., *Collected Works, Volume II: Publications from 1938-1974.* Oxford, Oxford Un. Press, pp.190-198, 1990 (original from 1949).

[9] B. Greene. *The Elegant Universe: Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory.* New York: W.W.Norton, 1999.

[10] W.S. Hatcher. *The Logical Foundations of Mathematics.* New York: Pergamon Press, 1982.

[11] W. Heisenberg. *Encounters with Einstein and Other Essays on People, Places, and Particles.* Princeton: Princeton Un. Press, 1989.

[12] T.J. Jech. About the axiom of choice. In J. Barwise (ed.), *Handbook of Mathematical Logic, pp. 345-70.*
newblockAmsterdam: North-Holland, 1977.

[13] D. Krause and J.R.B. Arenhart. *The Logical Foundations of Scientific Theories.* London: Routledge, 2017.

[14] D. Krause and J.R.B. Arenhart. Remarks on the use of formal methods. In Freund, M.A., Fernández, M. and Ruffino, M. (eds.), *The Logic and Philosophy of Logic: Recent Trends in Latin America and Spain.* London: College Pu, (Studies in Logic, 78), 2018, pp.181-202.

[15] G. Kreisel. Informal rigour and completeness proof. *Studies in Logic and the Foundations of Mathematics* 47, 1967, pp. 138-186.

[16] I. Lakatos. *Proofs and Refutations: The Logic of Mathematical Discovery.* Cambridge: Cambridge Un. Press, 1976.

[17] G.W. Mackey. *Mathematical Foundations of Quantum Mechanics.* New York: Dover (Dover Books on Mathematics), 2004.

[18] S. Mac Lane. *Categories for the Working Mathematician.* Graduate Texts in Mathematics. 5 (Second ed.). Springer, 1998.

[19] J.D. Maitland Wright, J. D. All Operators on a Hilbert Space are Bounded. *Bulletin of the American Mathematical Society 79(6), 1973, pp.1247-51.*

[20] A.P. Morse. *A Theory of Sets.* New York and London: Academic Press, 1965.

[21] J. Ortega y Gasset. *Obras Completas, Tomo II.* Madrid: Revista de Occidente Sexta Edición, 1963.

[22] J. Ortega y Gasset. *Meditaciones del Quijote.* Madrid: Publicaciones de la Residencia de Estudiantes, Serie II, Vol. I, 1914.

[23] R. Penrose. *The Road to Reality: A Complete Guide to the Laws of the Universe.* Vintage Books, 2004.

[24] K.R. Popper. *A Pocket Popper*. Edited by David Miller. Glasgow: Fontana Paperbacks, 1983.

[25] K.R. Popper. *Objective Knowledge: An Evolutionary Approach.* Oxford: Oxford Un. Press, Revised Edition, 1979.

[26] C. Rovelli. *Reality Is Not What It Seems: The Journey to Quantum Gravity* New York: Riverhear Books, 2017.

[27] L.-H. Robert. Quantum field theory: the Wightman axioms and the Haag-Kastler Axioms. http://www.unige.ch/math/folks/robert/zmpqft.pdf

[28] J. Shaw. Feyerabend and the cranks: on demarcation, epistemic virtues, and astrology. *Social Epistemology Review and Reply Collective* 6 (3), 2017, pp.74-88.

[29] R.M. Solovay. A model of set-theory in which every set of reals is Lebesgue measurable". *Annals of Mathematics.* Second Series 92: 1-56, 1970.

[30] P. Suppes. Representation and Invariance of Scientific Theories. Stanford: CSLI Pu., 2002

[31] B. van Fraassen. *The Scientific Image.* Oxford: Oxford Un. Press, 1980.

[32] M.B. Williams. Deducing the consequences of evolution: A mathematical model. *Journal of Theoretical Biology* 29 (3), 1979, pp. 343-85.

Décio Krause
Department of Philosophy
Federal University of Santa Catarina (UFSC)
C. P. 476
88040-900, Florianópolis, SC, Brazil
E-mail: deciokrause@gmail.com