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# Term Functor Logic Tableaux

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**Abstract**

The plus-minus calculus of Term Functor Logic features a peculiar algebra that, as of today, has not been used to produce a full tableaux method: here we offer one.

**Keywords:** Term logic, syllogistic, semantic tree.

## Introduction

Logic is about inference and in order to study it we usually make use of first order languages: first order logic and first order logic with identity are logical systems defined after first order languages. The origin of this habit has a complex and interesting history [10], but it is certainly related to the representative advantages first order languages offer when compared to traditional or Aristotelian term logic. By 1860, Augustus De Morgan had already pointed out the inability of Aristotelian term logic to deal with relations [9], but it was Russell who, around 1900, made popular the idea that the limits of the traditional logic programme, i.e. syllogistic, were due to a commitment to a ternary syntax, that is, a grammar of triads composed by a subject term and a predicate term joined by a copula [31]. Later, in 1930, Carnap would generalize this judgment to all traditional logic by claiming that its available syntax was predicative only, as in "All (some) Greeks are (not) mortal" or "Socrates is (not) mortal" [3].

It is true that the syntactical shortcomings of term logic cause big troubles when trying to represent propositions other than predicative, say relational (e.g., "Socrates and Plato are friends"), singular (e.g., "Socrates is mortal"), or compound (e.g., "If you are Socrates, you are Plato's friend"), but the major problem ternary syntax creates is term homogeneity. Geach argues:

> Our distinction between names and predicables enables us to clear up the confusion, going right back to Aristotle, as to whether

> there are genuine negative terms: predicables come in contradictory pairs, but names do not, and if names and predicables are both called "terms" there will be a natural hesitation over the question "Are there negative terms?" [17, p.64]

Broadly speaking, the worry with term homogeneity is that it does not allow us to preserve the fundamental noun-verb distinction. This incapacity is troublesome because the roles of nouns and predicates are not interchangeable: the function of a noun is naming, whereas the function of a predicate is predicating. Thus, the interchange of subject and predicate terms is a syntactical matter that produces an undesired semantic effect, for only a noun can be a logical subject, but a noun cannot maintain its role as a noun if it suddenly becomes a predicate. So, this syntactical issue turns out to be a semantic impossibility: between an Aristotelian term logic and genuine logic, goes Geach [17, p.54], there can only be war!

By contrast, genuine logic, namely first order logic, follows the Fregean paradigm that results from dropping terms and adopting a binary grammar of function-argument pairs. These pairs promote a syntax that includes individual constants $(a, b, c, \ldots)$ or variables $(x, y, z, \ldots)$ as arguments that stand for individual objects as logical subjects, plus relations $(A, B, C, \ldots)$ as functions that stand for concepts, not objects, as logical predicates. Thus, for instance, a singular proposition like "Socrates is mortal" could not be understood as a relation between a subject term and a predicate term, but as a function-argument pair were a constant, a saturated and complete element, say $s$, denotes an object named "Socrates" and works as an argument for the unsaturated and incomplete expression "... is mortal", say $Mx$, in such a way that the formula $Ms$ represents the proposition "Socrates is mortal": clearly, this last representation forbids any term shifting. Moreover, given this binary syntax, propositions like "All men are mortal" and "Every circle is a figure; therefore, anyone who draws a circle draws a figure" cannot be grasped as strings of terms but as strings of quantifiers, variables, and relations, say $\forall x(Hx \Rightarrow Mx)$ and $\forall x(Cx \Rightarrow Fx) \vdash \forall x((Dx \wedge \exists y(Cy \wedge Rxy)) \Rightarrow (Dx \wedge \exists y(Fy \wedge Rxy)))$, respectively.

Genuine logic, thus, results from discarding the ternary syntax (subject-copula-predicate) in order to favor a binary syntax (function-argument) that promotes the use of first order languages. This syntactical standard is of course familiar to us because we usually follow it when we teach, research, or apply logic: this is the received view of logic. However, it comes as no surprise that this view might very well feel familiar, but it is certainly not natural. Woods comments (emphasis is ours):

> It is no secret that classical logic and its mainstream variants

> aren't much good for human inference as it actually plays out in the conditions of real life—in life on the ground, so to speak. It isn't surprising. *Human reasoning is not what the modern orthodox logics were meant for.* The logics of Frege and Whitehead & Russell were purpose-built for the pacification of philosophical perturbation in the foundations of mathematics, notably but not limited to the troubles occasioned by the paradox of sets in their application to transfinite arithmetic. [40, p.404]

Genuine logic (classical, according to Woods), no doubt, has been fundamental for the study of inference, but it amazes us that, despite its original purpose, it is constantly used as a *bona fide* tool for representing natural language reasoning. Let us consider, to this effect, "Bar-Hillel's challenge" (emphasis is ours):

> *I challenge anybody here to show me a serious piece of argumentation in natural languages that has been successfully evaluated as to its validity with the help of formal logic.* I regard this fact as one of the greatest scandals of human existence. Why has this happened? How did it come to be that logic which, at least in the views of some people 2,300 years ago, was supposed to deal with evaluation of argumentation in natural languages, has done a lot of extremely interesting and important things, but not this? [36, p.256]

Since the late 60's, Fred Sommers championed a revision of the ternary syntax under the veil of Bar-Hillel's challenge, that is, under the assumption that logic has to deal with natural language reasoning. His project unfolded into three branches: ontology, semantics, and logic (cf. [34]). These branches became, respectively, a theory of categories that uses terms as the foundational elements of language, a theory of truth that employs the properties of terms in order to revive a correspondence theory of truth, and a theory of logic known as Term Functor Logic that takes terms as basic units of predication [32, 33, 35, 11, 14, 15].

This last theory is basically a plus-minus algebra—a "logibra", as Sommers dubbed it—that uses terms rather than first order language elements such as individual variables or quantifiers. However, since the proof methods for this theory are still in the making, in this contribution we offer a tableaux method for it. This goal should be of interest because the plus-minus calculus of Term Functor Logic features a peculiar algebra that, as of today, has not been used to produce a full tableaux method (cf. [8, 28]).[1] To reach this goal we briefly present Term Functor Logic (with special emphasis on syllogistic), then we introduce our contribution and, at the end, we discuss some of its features.

# 1 Preliminaries

## 1.1 Syllogistic

*Syllogistic* is a term logic that has its origins in Aristotle's *Prior Analytics* [1] and deals with inference between categorical propositions. A *categorical proposition* is a proposition composed by two terms, a quantity, and a quality. The subject and the predicate of a proposition are called *terms*: the term-schema S denotes the subject term of the proposition and the term-schema P denotes the predicate. The *quantity* may be either universal (*All*) or particular (*Some*) and the *quality* may be either affirmative (*is*) or negative (*is not*). These categorical propositions have a *type* denoted by a label (either a (universal affirmative, SaP), e (universal negative, SeP), i (particular affirmative, SiP), or o (particular negative, SoP)) that allows us to determine a *mood*, that is, a sequence of three categorical propositions ordered in such a way that two propositions are premises and the last one is a conclusion. A *categorical syllogism*, then, is a mood with three terms one of which appears in both premises but not in the conclusion. This particular term, usually denoted with the term-schema M, works as a link between the remaining terms and is known as the middle term. According to the position of this middle term, four *figures* can be set up in order to encode the valid syllogistic moods (Table 1).[2]

| First Figure | Second Figure | Third Figure | Fourth Figure |
|---|---|---|---|
| aaa | eae | iai | aee |
| eae | aee | aii | iai |
| aii | eio | oao | eio |
| eio | aoo | eio | |

Table 1: Valid syllogistic moods

[1][35, p.183ff] have already advanced a proposal, but its scope is limited to propositional logic.

[2]For sake of brevity, but without loss of generality, here we omit the syllogisms that require existential import.

## 1.2 Term Functor Logic

*Term Functor Logic* (TFL) is a plus-minus calculus, developed by Sommers [32, 33, 35] and Englebretsen [11, 14, 15], that deals with syllogistic by using terms rather than first order language elements such as individual variables or quantifiers.[3] According to this algebra, the four categorical propositions can be represented by the following syntax:[4]

- $\mathsf{SaP} := -\mathsf{S} + \mathsf{P} = -\mathsf{S} - (-\mathsf{P}) = -(-\mathsf{P}) - \mathsf{S} = -(-\mathsf{P}) - (+\mathsf{S})$
- $\mathsf{SeP} := -\mathsf{S} - \mathsf{P} = -\mathsf{S} - (+\mathsf{P}) = -\mathsf{P} - \mathsf{S} = -\mathsf{P} - (+\mathsf{S})$
- $\mathsf{SiP} := +\mathsf{S} + \mathsf{P} = +\mathsf{S} - (-\mathsf{P}) = +\mathsf{P} + \mathsf{S} = +\mathsf{P} - (-\mathsf{S})$
- $\mathsf{SoP} := +\mathsf{S} - \mathsf{P} = +\mathsf{S} - (+\mathsf{P}) = +(-\mathsf{P}) + \mathsf{S} = +(-\mathsf{P}) - (-\mathsf{S})$

Given this algebraic representation, the plus-minus algebra offers a simple method of decision for syllogistic: a conclusion follows validly from a set of premises if and only if *i)* the sum of the premises is algebraically equal to the conclusion and *ii)* the number of conclusions with particular quantity (viz., zero or one) is the same as the number of premises with particular quantity [14, p.167]. Thus, for instance, if we consider a valid syllogism, say the mood aaa from figure 1, we can see how the application of this method produces the right conclusion (Table 2).

| | Proposition | TFL |
|---|---|---|
| 1. | All dogs are animals. | $-\mathsf{D} + \mathsf{A}$ |
| 2. | All German Shepherds are dogs. | $-\mathsf{G} + \mathsf{D}$ |
| $\vdash$ | All German Shepherds are animals. | $-\mathsf{G} + \mathsf{A}$ |

Table 2: A valid syllogism: aaa-1

In the previous example we can clearly see how the method works: *i)* if we add up the premises we obtain the algebraic expression $(-\mathsf{D} + \mathsf{A}) + (-\mathsf{G} + \mathsf{D}) = -\mathsf{D} + \mathsf{A} - \mathsf{G} + \mathsf{D} = -\mathsf{G} + \mathsf{A}$, so that the sum of the premises is algebraically equal to the conclusion and the conclusion is $-\mathsf{G} + \mathsf{A}$, rather than $+\mathsf{A} - \mathsf{G}$,

[3] That we can represent and perform inference without first order language elements such as individual variables or quantifiers is not news (cf. [29, 25, 20]), but Sommers' logical project has a wider impact: that we can use a logic of terms instead of a first order system has nothing to do with the mere syntactical fact, as it were, that we can reason without quantifiers or variables, but with the general view that natural language is a source of natural logic (cf. [33, 34, 22]).

[4] We mainly focus on the presentation by [14].

because *ii)* the number of conclusions with particular quantity (zero in this case) is the same as the number of premises with particular quantity (zero in this case).

This algebraic approach is also capable of representing relational, singular, and compound propositions with ease and clarity while preserving its main idea, namely, that inference is a logical procedure between terms. For example, the following cases illustrate how to represent and perform inference with relational (Table 3), singular[5] (Table 4), or compound propositions[6] (Table 5).

| | Proposition | TFL |
|---|---|---|
| 1. | Some horses are faster than some dogs. | $+\mathsf{H}_1 + (+\mathsf{F}_{12} + \mathsf{D}_2)$ |
| 2. | Dogs are faster than some men. | $-\mathsf{D}_2 + (+\mathsf{F}_{23} + \mathsf{M}_3)$ |
| 3. | That which is faster than what is faster than men, is faster than men. | $-(+\mathsf{F}_{12} + (+\mathsf{F}_{23} + \mathsf{M}_3))+$ $(+\mathsf{F}_{13} + \mathsf{M}_3)$ |
| ⊢ | Some horses are faster than some men. | $+\mathsf{H}_1 + (+\mathsf{F}_{13} + \mathsf{M}_3)$ |

Table 3: A valid inference with relational propositions

| | Proposition | TFL |
|---|---|---|
| 1. | All men are mortal. | $-\mathsf{M} + \mathsf{L}$ |
| 2. | Socrates is a man. | $+\mathsf{s} + \mathsf{M}$ |
| ⊢ | Socrates is mortal. | $+\mathsf{s} + \mathsf{L}$ |

Table 4: A valid inference with singular propositions

| | Proposition | TFL |
|---|---|---|
| 1. | If you are Socrates, you are Plato's friend. | $-[\mathsf{s}] + [\mathsf{p}]$ |
| 2. | You are Socrates. | $+[\mathsf{s}]$ |
| ⊢ | You are Plato's friend. | $+[\mathsf{p}]$ |

Table 5: A valid inference with compound propositions

These examples are designed to show that TFL is capable of dealing with a wide range of inferences, namely, those classical first order logic is capable of dealing with. However, in certain sense, TFL is arguably more expressive than

[5]Provided singular terms, such as *Socrates*, are represented by lowercase letters.

[6]Given that compound propositions can be represented as follows, $P := +[p]$, $Q := +[q]$, $\neg P := [-p]$, $P \Rightarrow Q := -[p] + [q]$, $P \wedge Q := +[p] + [q]$, and $P \vee Q := --[p] - -[q]$, the method of decision behaves like resolution (cf. [26]).

classical first order logic. Let us consider, for example, the following natural language inference [11, p.172]:

*Plato taught Aristotle. So, Aristotle was taught by Plato.*

It seems fair to say that the previous inference is a valid one, after all, it is impossible for the premises to be true and the conclusion to be false. However, it is not clear how such an inference is valid in classical first order logic. Consider, for sake of explanation, the following first order representation:

$$Tpa \vdash Tpa$$

In the original example it is evident the conclusion is semantically equivalent to the premise, yet the premise is syntactically different to the conclusion. Ideally, this semantic difference should also be a syntactic one, for even if we recognize that the premise and the conclusion share the same meaning, active voice and passive voice are not syntactically equivalent. Now, classical first order logic avoids this issue by assuming that such a syntactic difference is not relevant because both propositions share the same propositional content, but this strikes us as an *ad hoc* solution because, for instance, commutative pairs of propositions also share the same propositional content yet we do not disregard the syntactic difference. By contrast, TFL is capable of preserving the semantic equivalence while indicating the syntactic difference, thus being able to perform inference with active-passive voice transformations (for instance, by applying *Com* and *Assoc* (for a summary of these rules *vide* Appendix A)):

$$+\mathsf{p}_1 + (+\mathsf{T}_{12} + \mathsf{a}_2) \vdash +\mathsf{a}_2 + (+\mathsf{T}_{12} + \mathsf{p}_1)$$

Further, consider another valid inference [11, p.173]:

*Socrates taught a teacher of Aristotle. So, one whom Socrates taught, taught Aristotle.*

A plausible representation of the previous inference in classical first order logic would be:

$$\exists x(Tsx \wedge Txa) \vdash \exists x(Tsx \wedge Txa)$$

But again, this representation does not seem to be a faithful transcription because it is unable to preserve a subtle but meaningful difference between the premise and the conclusion, namely, the associative shift. By contrast, TFL is able to perform inference with associative shifts (for instance, by applying *Assoc*):

$$+\mathsf{s}_1 + (+\mathsf{T}_{12} + (+\mathsf{T}_{23} + \mathsf{a}_3)) \vdash +(+\mathsf{s}_1 + \mathsf{T}_{12}) + (+\mathsf{T}_{23} + \mathsf{a}_3)$$

Last but not least, consider another valid inference [11, p.174]:

*Plato taught Aristotle with a dialogue. So, Plato taught Aristotle.*

A possible representation of the previous inference in classical first order logic could be:

$$\exists x(Dx \wedge Tpax) \vdash Tpa$$

Now, since the relations $Txyz$ and $Txy$ have different arity, such an intuitive inference is not valid *prima facie*, and so, for it to be valid in classical first order logic we need an extralogical justification capable of connecting both relations. By contrast, TFL is able to perform inference with polyadic simplifications: it preserves the validity of the previous inference under the common sense assumption that the *teaching* relation is polyadic (for example, by applying *Assoc* and *Simp*):

$$+\mathsf{p}_1 + ((+\mathsf{T}_{123} + \mathsf{a}_2) + \mathsf{D}_3) \vdash +\mathsf{p}_1 + (+\mathsf{T}_{123} + \mathsf{a}_2)$$

We will return to these features later.

## 2 TFL tableaux

As we can see, the peculiar algebra of TFL has some interesting capacities (and inference rules); however, as of today, this algebra has not been exploited as to produce a full tableaux method (cf. [8, 35, 28]): so here we propose one in three steps. First, we start by offering some rules; then we show how we can apply those rules in three different inferential contexts (basic syllogistic, relational syllogistic, and propositional logic); and finally, we offer some evidence to the effect that the method is reliable.

As usual, and following [8, 28], we say a *tableau* is an acyclic connected graph determined by nodes and vertices. The node at the top is called *root*. The nodes at the bottom are called *tips*. Any path from the root down a series of vertices is a *branch*. To test an inference for validity we construct a tableau which begins with a single branch at whose nodes occur the premises and the rejection of the conclusion: this is the *initial list*. We then apply the rules that allow us to extend the initial list: Diagram 1

In Diagram 1, from left to right, the first rule is the rule for a (e) propositions, and the second rule is the rule for i (o) propositions. Notice that, after applying a rule, we introduce some index $i \in \{1, 2, 3, \ldots\}$. For propositions


Diagram 1: TFL tableaux rules

a and e, the index may be any number; for propositions i and o, the index has to be a new number if they do not already have an index. Also, following TFL tenets, we assume the followings rules of rejection: $-(\pm \mathsf{A}) = \mp \mathsf{A}$, $-(\pm \mathsf{A} \pm \mathsf{B}) = \mp \mathsf{A} \mp \mathsf{B}$, and $-(- - \mathsf{A} - -\mathsf{A}) = +(-\mathsf{A}) + (-\mathsf{A})$.

As usual, a tableau is *complete* if and only if every rule that can be applied has been applied. A branch is *closed* if and only if there are terms of the form $\pm \mathsf{A}^i$ and $\mp \mathsf{A}^i$ on two of its nodes; otherwise it is *open*. A closed branch is indicated by writing a $\perp$ at the end of it; an open branch is indicated by writing $\infty$. A tableau is *closed* if and only if every branch is closed; otherwise it is *open*. So, again as usual, A is a logical consequence of the set of terms $\Gamma$ (i.e., $\Gamma \vdash \mathsf{A}$) if and only if there is a complete closed tableau whose initial list includes the terms of $\Gamma$ and the rejection of A (i.e., $\Gamma \cup \{-\mathsf{A}\} \vdash \perp$).

Accordingly, up next we show the method works for basic syllogistic (Diagram 2) and relational syllogistic (Table 6, Diagram 3), including cases of active-passive voice transformations, associative shifts, and polyadic simplifications (Table 7, Diagram 4).

| | Proposition | TFL |
|---|---|---|
| 1. | Every boy loves some girl. | $-\mathsf{B}_1 + (+\mathsf{L}_{12} + \mathsf{G}_2)$ |
| 2. | Every girl adores some cat. | $-\mathsf{G}_1 + (+\mathsf{A}_{12} + \mathsf{C}_2)$ |
| 3. | All cats are mangy. | $-\mathsf{C} + \mathsf{M}$ |
| 4. | Whoever adores something mangy is a fool. | $-(+\mathsf{A}_{12} + \mathsf{M}_1) + \mathsf{F}_2$ |
| $\vdash$ | Every boy loves something fool. | $-\mathsf{B}_1 + (+\mathsf{L}_{12} + \mathsf{F}_2)$ |

Table 6: A relational syllogistic example (adapted from [14, p.172])

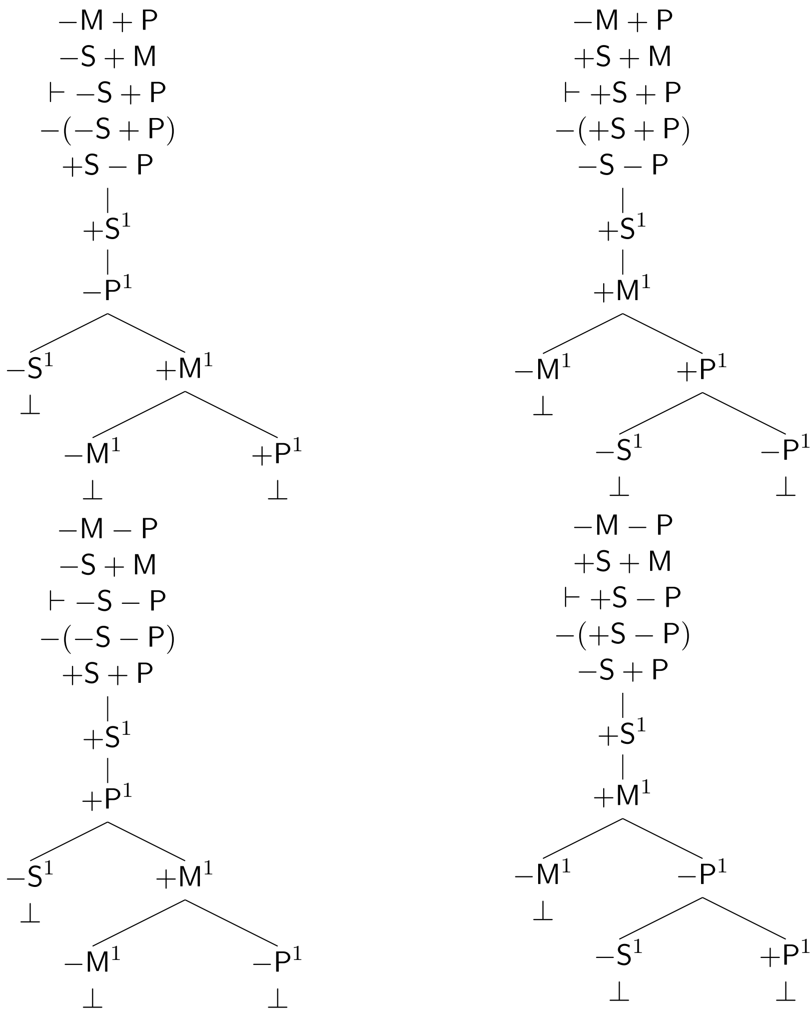



Diagram 2: Moods aaa-1, eae-1, aii-1, and eio-1

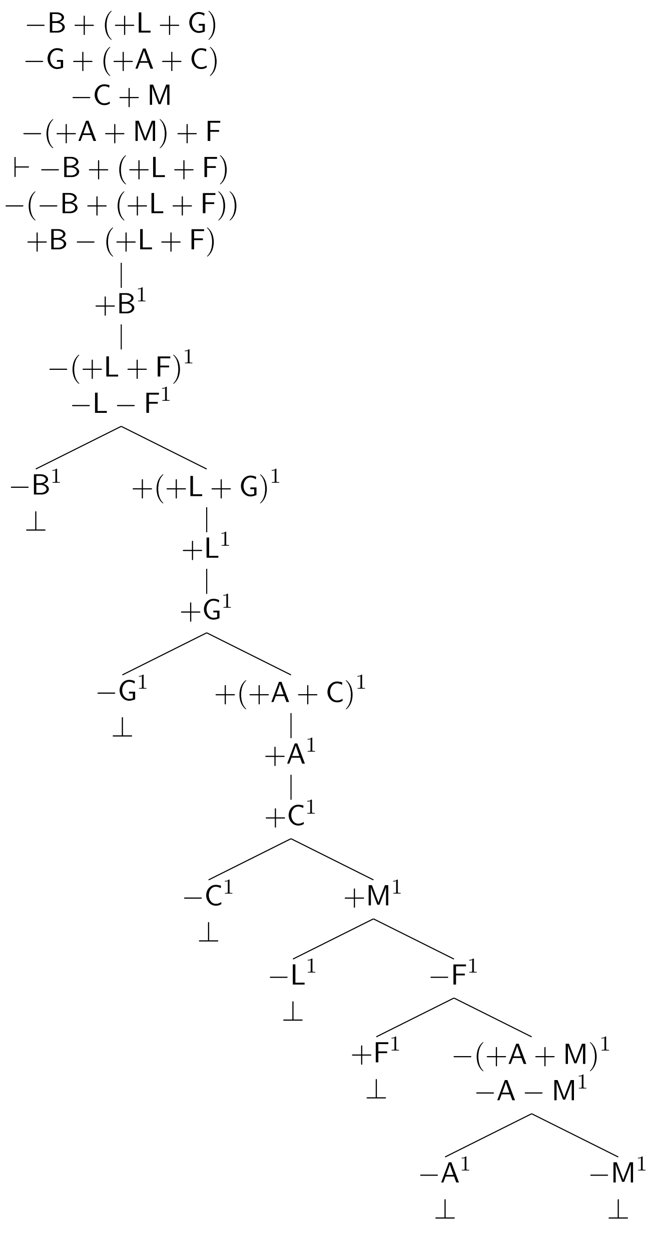



Diagram 3: A relational syllogistic tableau

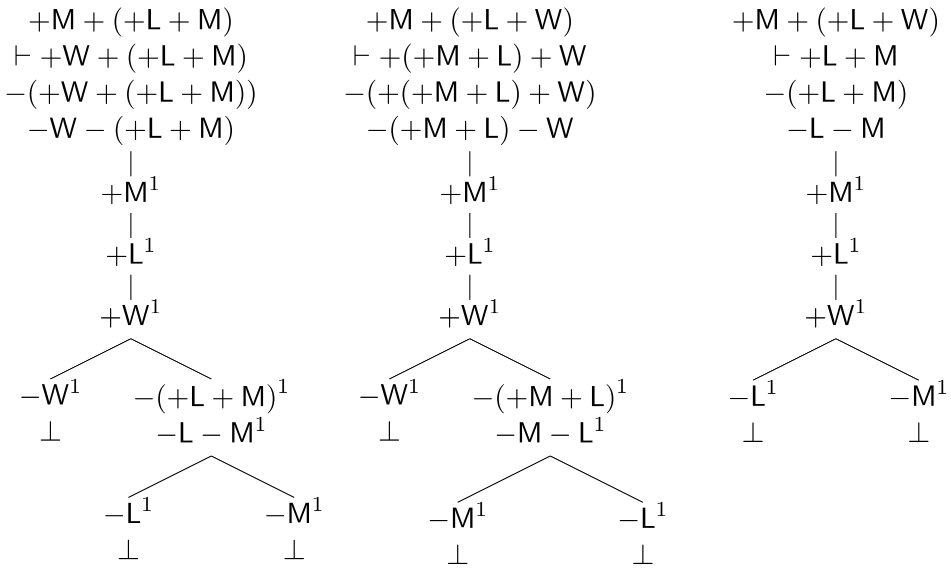


Diagram 4: Passive-active voice transformation, associative shift, and polyadic simplification

| | Proposition | TFL |
|---|---|---|
| 1. | Some man loves some woman. | $+\mathsf{M}_1 + (+\mathsf{L}_{12} + \mathsf{W}_2)$ |
| 2. | What a man loves is a woman. | $+(+\mathsf{M}_1 + \mathsf{L}_{12}) + \mathsf{W}_2$ |
| 3. | A woman is something a man loves. | $+\mathsf{W}_2 + (+\mathsf{M}_1 + \mathsf{L}_{12})$ |
| 4. | A woman is loved by a man. | $+\mathsf{W}_2 + (+\mathsf{L}_{12} + \mathsf{M}_1)$ |
| 5. | Some lover is a man. | $+\mathsf{L}_{12} + \mathsf{M}_1$ |

Table 7: Passive-active voice transformation, associative shift, and polyadic simplification examples (adapted from [14, p.174])

As a sidenote, we now mention how the method can be used for propositional logic. Recall the transcription from propositional logic to TFL is as follows: $P := +[p]$, $Q := +[q]$, $\neg P := -[p]$, $P \Rightarrow Q := -[p] + [q]$, $P \wedge Q := +[p] + [q]$, and $P \vee Q := --[p] - -[q]$, the rules, then, go as in Diagram 5 (notice that for the propositional case we need not use the superscript indexes).

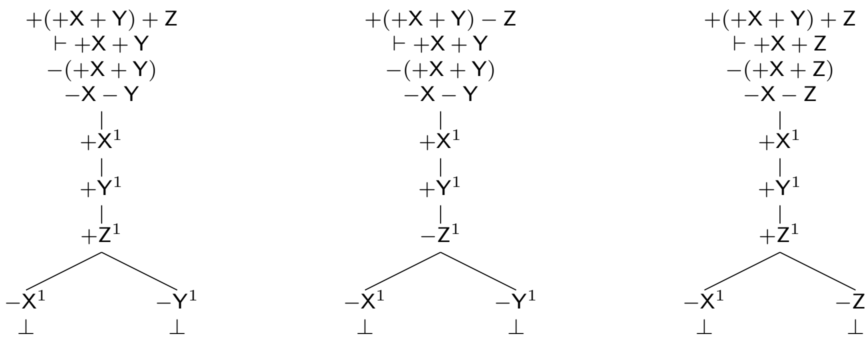

Diagram 5: TFL tableux rules for propositional logic

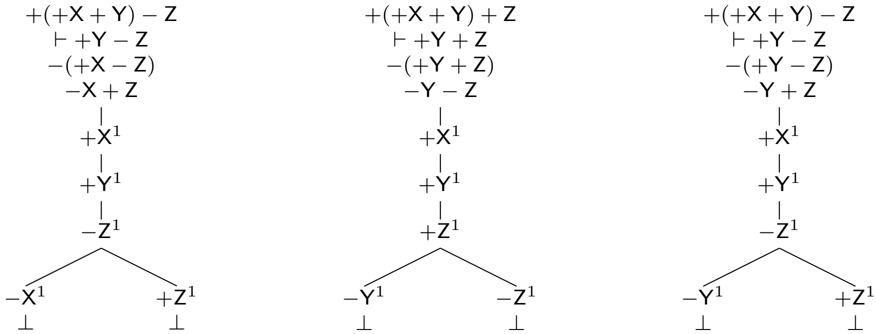
## 2.1 Reliability

Finally, we produce some evidence to the effect that this method is reliable (*zuverlässig*) in the sense that what can be proven using the inference rules (say, $\mathsf{TFL}^{\mathsf{rules}}_{\mathsf{valid}}$) produces closed complete tableaux (say, $\mathsf{TFL}^{\mathsf{tableaux}}_{\mathsf{valid}}$), and vice versa. For the purposes of this study we need only focus on mediate inference rules.

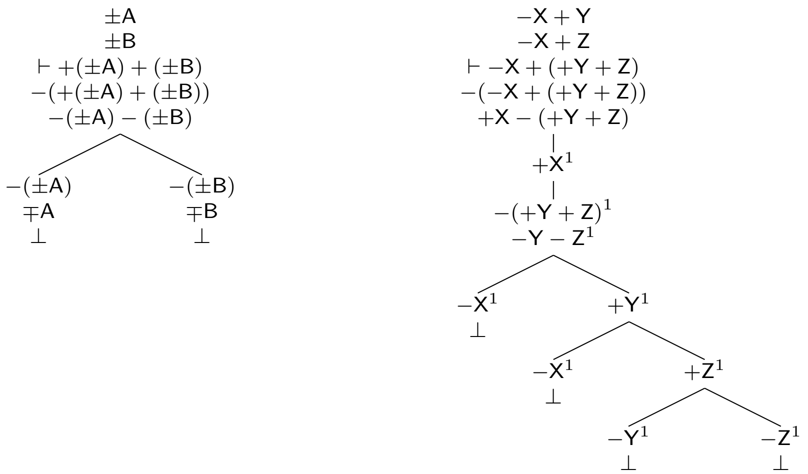
**Proposition 2.1** *If an inference is* $\mathsf{TFL}^{\mathsf{rules}}_{\mathsf{valid}}$*, it is* $\mathsf{TFL}^{\mathsf{tableaux}}_{\mathsf{valid}}$*.*

**Proof.** We proceed by cases. We check each mediate inference rule of TFL (*vide* Appendix) is $\mathsf{TFL}^{\mathsf{tableaux}}_{\mathsf{valid}}$. For the rule *DON* we only need to retort to Section 3.1: in there we can observe the four possible ocurrences of *DON* and how they are $\mathsf{TFL}^{\mathsf{tableaux}}_{\mathsf{valid}}$. For the rule *Simp* we can build the next tableaux and observe they are all $\mathsf{TFL}^{\mathsf{tableaux}}_{\mathsf{valid}}$:






Finally, for the rule *Add* we can build the next tableaux:




■

**Proposition 2.2** *If an inference is* $\mathsf{TFL}^{\mathsf{tableaux}}_{\mathsf{valid}}$*, it is* $\mathsf{TFL}^{\mathsf{rules}}_{\mathsf{valid}}$.

**Proof.** We proceed by *reductio*. Suppose we pick an arbitrary inference that is $\mathsf{TFL}^{\mathsf{tableaux}}_{\mathsf{valid}}$ but is not $\mathsf{TFL}^{\mathsf{rules}}_{\mathsf{valid}}$. Then there is a complete closed tableau whose initial list includes the set of terms $\Gamma$ (possibly empty) and the rejection of the conclusion; but from $\Gamma$ alone we cannot construct a proof of the conclusion by using any of the rules of mediate inference. There are three general cases using the tableaux rules: a complete closed tableau whose conclusion is $-\mathsf{A} \pm \mathsf{B}$, $+\mathsf{A} \pm \mathsf{B}$, or $-\mathsf{A} + \mathsf{A}$ when $\Gamma$ is empty. Since in each case the tableau is complete, the corresponding rules have been applied; and since each tableau is closed, each tableau must be of one of the following general forms:

$$
\begin{array}{c}
\Gamma \\
\vdash -\mathsf{A} \pm \mathsf{B} \\
-(-\mathsf{A} \pm \mathsf{B}) \\
+\mathsf{A} \mp \mathsf{B} \\
| \\
+\mathsf{A} \\
| \\
\mp\mathsf{B} \\
| \\
\vdots \\
\begin{array}{cc}
-\mathsf{A} \in \Gamma & \pm\mathsf{B} \in \Gamma \\
\bot & \bot
\end{array}
\end{array}
\qquad
\begin{array}{c}
\Gamma \\
\vdash +\mathsf{A} \pm \mathsf{B} \\
-(+\mathsf{A} \pm \mathsf{B}) \\
-\mathsf{A} \mp \mathsf{B} \\
| \\
+\mathsf{A} \in \Gamma \\
| \\
\pm\mathsf{B} \in \Gamma \\
| \\
\vdots \\
\begin{array}{cc}
-\mathsf{A} & \mp\mathsf{B} \\
\bot & \bot
\end{array}
\end{array}
\qquad
\begin{array}{c}
\vdash -\mathsf{A} + \mathsf{A} \\
-(-\mathsf{A} + \mathsf{A}) \\
+\mathsf{A} - \mathsf{A} \\
| \\
+\mathsf{A}^i \\
| \\
-\mathsf{A}^i \\
\bot
\end{array}
$$

So, suppose we have an instance of the first general form but the corresponding inference is not $\mathsf{TFL}^{\mathsf{rules}}_{\mathsf{valid}}$, i.e., where $\Gamma^+ = \Gamma \cup \{+\mathsf{A} \mp \mathsf{B}\}$, $\Gamma^+ \vdash \bot$, but from any application of *DON*, *Simp*, and *Add* to $\Gamma$, the conclusion $-\mathsf{A} \pm \mathsf{B}$ does not obtain. Now, by following the paths of the tableau of the first general form, we observe that, at the bottom, the tableau has a couple of closed branches. Hence, at some previous nodes the tableau has to include something of the form $-\mathsf{A} + \mathsf{X}, -\mathsf{X} \pm \mathsf{B}$, that is to say, we need $\Gamma = \{\ldots, -\mathsf{A} + \mathsf{X}, -\mathsf{X} \pm \mathsf{B}, \ldots\}$. But if so, by applying *DON*, we obtain $-\mathsf{A} \pm \mathsf{B}$ from $\Gamma$, which contradicts the assumption. Similarly, suppose we have an instance of the second general form but the corresponding inference is not $\mathsf{TFL}^{\mathsf{rules}}_{\mathsf{valid}}$, i.e., where $\Gamma^+ = \Gamma \cup \{-\mathsf{A} \mp \mathsf{B}\}$, $\Gamma^+ \vdash \bot$, but from any application of *DON*, *Simp*, and *Add* to $\Gamma$, the conclusion $+\mathsf{A} \pm \mathsf{B}$ does not obtain. By following the paths of the tableau of the second general form, we observe that, at the bottom, the tableau has a couple of closed branches, and for those branches to be closed, we need something of the form $+\mathsf{A}, \pm\mathsf{B}$ or something of the form $+(+\mathsf{A} + \mathsf{X}) \pm \mathsf{B}$ at some previous nodes of the tableau, that is to say, we need either $\Gamma = \{\ldots, +\mathsf{A}, \pm\mathsf{B}, \ldots\}$ or $\Gamma = \{\ldots, +(+\mathsf{A} + \mathsf{X}) \pm \mathsf{B}, \ldots\}$. But if so, in either case, by applying *Add*, we obtain $+\mathsf{A} \pm \mathsf{B}$ from $\Gamma$; and by applying *Simp*, we obtain $+\mathsf{A} \pm \mathsf{B}$ from $\Gamma$, which contradicts the assumption. Finally, assume we have an instance of the third general form. In that case, the path of the tableau consists only of $\Gamma^+ = \{+\mathsf{A} - \mathsf{A}\}$, and so, trivially, $\Gamma^+ \vdash \bot$. But since $\Gamma$ is empty, $-\mathsf{A} + \mathsf{A}$ has to be a tautology (i.e., *All* A *is* A) (cf. [14, p.168]). ■

# 3 Concluding remarks

In this contribution we have attempted to offer a full tableaux method for Term Functor Logic. Here are some remarks we can extract from this attempt: *a)* the tableaux method we have submitted avoids condition *ii)* of the method of decision for syllogistic (namely, that the number of particular premises has to be equal to the number of particular conclusions), thus allowing its general application for any number of premisses. *b)* The method preserves the power of TFL with respect to relational inferences, passive-active voice transformations, associative shifts, and polyadic simplifications, something that gives this method a competitive advantage over classical first order logic (tableaux). *c)* As a particular case, when no superscript index is used, we just obtain a tableaux method for propositional logic. *d)* Due to the peculiar algebra of TFL, we have no use for quantification rules nor skolemization, which could be useful for logic programming. *e)* The number of inference rules (cf. [14, p.168-170]) gets drastically reduced to a shorter, simpler, and uniform set of tableaux rules that preserves the capacity of TFL to perform inference in different inferential contexts (basic syllogistic, relational syllogistic, and propositional logic). *f)* Also, we have to mention that for the purposes of this paper we have focused only on the terministic features of TFL, but further comparison is required with the algebraic proof systems introduced by [6, 4], since these systems allow us to reconstruct Boole's analysis of syllogistic by employing polynomial formatted proofs [5] and can also be extended to several other logics, like modal logic [2, 7]. *g)* Finally, we need to add that, due to reasons of space, we are unable to introduce the modal [12, 30, 21], intermediate [27, 37] or numerical [24] extensions of the method that allow us to represent and reason with modal propositions or non-classical quantifiers; however, we need to stress that the inferential and representative powers of term logics go far beyond the limits of the traditional or first order logic frontiers (cf. [22]).

For all these reasons, we believe this proof procedure is not only novel, but also promising, not just as yet another critical thinking tool or didactic contraption, but as a research device for probabilistic and numerical reasoning (in so far as it could be used to represent probabilistic (cf. [38]) or numerical reasoning (cf. [24])), diagrammatic reasoning (as it finds its natural home in a project of visual reasoning (cf. [13, 35])), psychology (as it could be used to approximate a richer psychological account of syllogistic reasoning (cf. [19, 18])), artificial intelligence (as it could be used to develop tweaked inferential engines for Aristotelian databases (cf. [23])), and of course, philosophy of logic (as it promotes the revision and revival of term logic (cf. [39, 33, 14, 15]) as a tool that might be more interesting and powerful than once it seemed (cf. [3, 16, 17])). We are currently working on some of these areas.

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## References

[1] Aristotle. *Prior Analytics*. Hackett Classics Series. Hackett, 1989.

[2] Agudelo Juan C. and Walter A. Carnielli. Polynomial ring calculus for modal logics: A new semantics and proof method for modalities. *The Review of Symbolic Logic*, 4(1):150–170, 2011.

[3] Rudolf Carnap. Die alte und die neue logik. *Erkenntnis*, 1:12–26, 1930.

[4] Walter A. Carnielli. Polynomial ring calculus for many-valued logics. In *35th International Symposium on Multiple-Valued Logic (ISMVL'05)*, pages 20–25, 2005.

[5] Walter A. Carnielli. Polynomizing: Logic inference in polynomial format and the legacy of boole. In *Model-Based Reasoning in Science, Technology, and Medicine*, 2007.

[6] Walter A. Carnielli. A polynomial proof system for Łukasiewicz logics. In *Second Principia International Symposium*, August 6-10, 2001.

[7] Walter A. Carnielli and Mariana Matulovic. Non-deterministic semantics in polynomial format. *Electronic Notes in Theoretical Computer Science*, 305:19–34, 2014. Proceedings of the 8th Workshop on Logical and Semantic Frameworks (LSFA).

[8] Marcello D'Agostino, Dov M. Gabbay, Reiner Hähnle, and Joachim Posegga. *Handbook of Tableau Methods*. Springer, 1999.

[9] A. de Morgan. On the Syllogism, No. IV., and on the Logic of Relations. *Transactions of the Cambridge Philosophical Society*, 10:331, 1864.

[10] Matti Eklund. On how logic became first-order. *Nordic Journal of Philosophical Logic*, 1(2):147–67, 1996.

[11] George Englebretsen. *The New Syllogistic*. 05. P. Lang, 1987.

[12] George Englebretsen. Preliminary notes on a new modal syllogistic. *Notre Dame J. Formal Logic*, 29(3):381–395, 1988.

[13] George Englebretsen. Linear diagrams for syllogisms (with relationals). *Notre Dame J. Formal Logic*, 33(1):37–69, 1991.

[14] George Englebretsen. *Something to Reckon with: The Logic of Terms.* Canadian electronic library: Books collection. University of Ottawa Press, 1996.

[15] George Englebretsen and Charles Sayward. *Philosophical Logic: An Introduction to Advanced Topics.* Bloomsbury Academic, 2011.

[16] Peter T. Geach. *Reference and Generality: An Examination of Some Medieval and Modern Theories.* Contemporary Philosophy / Cornell University. Cornell University Press, 1962.

[17] Peter T. Geach. *Logic Matters.* Campus (University of California Press). University of California Press, 1980.

[18] Frank C. Keil. Exploring boundary conditions on the structure of knowledge: Some nonobvious influences of philosophy on psychology. In David S. Oderberg, editor, *The Old New Logic: Essays on the Philosophy of Fred Sommers*, pages 67–84. Bradford book, 2005.

[19] Sangeet Khemlani and Philip N. Johnson-Laird. Theories of the syllogism: a meta-analysis. *Psychological Bulletin*, pages 427–457, 2012.

[20] Steven T. Kuhn. An axiomatization of predicate functor logic. *Notre Dame J. Formal Logic*, 24(2):233–241, 1983.

[21] M. Malink. *Aristotle's Modal Syllogistic.* Harvard University Press, 2013.

[22] Larry Moss. Natural logic. In S. Lappin and C. Fox, editors, *The Handbook of Contemporary Semantic Theory.* John Wiley & Sons, 2015.

[23] Eyal Mozes. A deductive database based on aristotelian logic. *Journal of Symbolic Computation*, 7(5):487–507, 1989.

[24] Wallace A. Murphree. Numerical term logic. *Notre Dame J. Formal Logic*, 39(3):346–362, 1998.

[25] Aris Noah. Predicate-functors and the limits of decidability in logic. *Notre Dame J. Formal Logic*, 21(4):701–707, 1980.

[26] Aris Noah. Sommers's cancellation technique and the method of resolution. In David S. Oderberg, editor, *The Old New Logic: Essays on the Philosophy of Fred Sommers*, pages 169–182. Bradford, 2005.

[27] Philip L. Peterson. On the logic of "few", "many", and "most". *Notre Dame J. Formal Logic*, 20(1):155–179, 1979.

[28] Graham Priest. *An Introduction to Non-Classical Logic: From If to Is*. Cambridge Introductions to Philosophy. Cambridge University Press, 2008.

[29] Willard Van Orman Quine. Predicate functor logic. In J E Fenstad, editor, *Proceedings of the Second Scandinavian Logic Symposium*. North-Holland, 1971.

[30] Adriane A. Rini. Is there a modal syllogistic? *Notre Dame J. Formal Logic*, 39(4):554–572, 1998.

[31] B. Russell. *A Critical Exposition of the Philosophy of Leibniz: With an Appendix of Leading Passages*. Cambridge University Press.

[32] Fred Sommers. On a fregean dogma. In Imre Lakatos, editor, *Problems in the Philosophy of Mathematics*, volume 47 of *Studies in Logic and the Foundations of Mathematics*, pages 47–81. Elsevier, 1967.

[33] Fred Sommers. *The Logic of Natural Language*. Clarendon Library of Logic and Philosophy. Clarendon Press; Oxford: New York: Oxford University Press, 1982.

[34] Fred Sommers. Intelectual autobiography. In David S. Oderberg, editor, *The Old New Logic: Essays on the Philosophy of Fred Sommers*, pages 1–24. Bradford book, 2005.

[35] Fred Sommers and George Englebretsen. *An Invitation to Formal Reasoning: The Logic of Terms*. Ashgate, 2000.

[36] J. F. Staal. Formal logic and natural languages (a symposium). *Foundations of Language*, 5(2):256–284, 1969.

[37] Bruce Thompson. Syllogisms using "few", "many", and "most". *Notre Dame J. Formal Logic*, 23(1):75–84, 1982.

[38] Bruce Thompson. Syllogisms with statistical quantifiers. *Notre Dame J. Formal Logic*, 27(1):93–103, 1986.

[39] Henry Babcock Veatch. *Intentional logic: a logic based on philosophical realism*. Archon Books, 1970.

[40] John Woods. *Logic Naturalized*, pages 403–432. Springer International Publishing, Cham, 2016.

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## Appendix A. Rules of inference for TFL

In this Appendix we expound the rules of inference for TFL as they appear in [14, p.168-170].

### Rules of immediate inference

1. Premise ($P$): Any premise or tautology can be entered as a line in proof. (Tautologies that repeat the corresponding conditional of the inference are excluded. The corresponding conditional of an inference is simply a conditional sentence whose antecedent is the conjunction of the premises and whose consequent is the conclusion.)

2. Double Negation ($DN$): Pairs of unary minuses can be added or deleted from a formula (i.e., $- - X = X$).

3. External Negation ($EN$): An external unary minus can be distributed into or out of any phrase (i.e., $-(\pm X \pm Y) = \mp X \mp Y$).

4. Internal Negation ($IN$): A negative qualifier can be distributed into or out of any predicate-term (i.e.,$\pm X - (\pm Y) = \pm X + (\pm Y)$).

5. Commutation ($Com$): The binary plus is symmetric (i.e., $+X + Y = +Y + X$).

6. Association ($Assoc$): The binary plus is associative (i.e., $+X + (+Y + Z) = +(+X + Y) + Z$).

7. Contraposition ($Contrap$): The subject- and predicate-terms of a universal affirmation can be negated and can exchange places (i.e., $-X + Y = -(-Y) + (-X)$).

8. Predicate Distribution ($PD$): A universal subject can be distributed into or out of a conjunctive predicate (i.e., $-X + (+Y + Z) = +(-X + Y) + (-X + Z)$) and a particular subject can be distributed into or out of a disjunctive predicate (i.e., $+X + (-(-Y) - (-Z)) = - - (+X + Y) - -(+X + Z)$).

9. Iteration ($It$): The conjunction of any term with itself is equivalent to that term (i.e., $+X + X = X$).

### Rules of mediate inference

1. ($DON$): If a term, $M$, occurs universally quantified in a formula and either $M$ occurs not universally quantified or its logical contrary occurs

universally quantified in another formula, deduce a new formula that is exactly like the second except that $M$ has been replaced at least once by the first formula minus its universally quantified $M$.

2. Simplification (*Simp*): Either conjunct can be deduced from a conjunctive formula; from a particularly quantified formula with a conjunctive subject-term, deduce either the statement form of the subject-term or a new statement just like the original but without one of the conjuncts of the subject-term (i.e., from $+(+X+Y) \pm Z$ deduce any of the following: $+X+Y$, $+X \pm Z$, or $+Y \pm Z$), and from a universally quantified formula with a conjunctive predicate- term deduce a new statement just like the original but without one of the conjuncts of the predicate-term (i.e., from $-X \pm (+Y + Z)$ deduce either $-X \pm Y$ or $-X \pm Z$).

3. Addition (*Add*): Any two previous formulae in a sequence can be conjoined to yield a new formula, and from any pair of previous formulae that are both universal affirmations and share a common subject-term a new formula can be derived that is a universal affirmation, has the subject-term of the previous formulae, and has the conjunction of the predicate- terms of the previous formulae as its predicate-term (i.e., from $-X + Y$ and $-X + Z$ deduce $-X + (+Y + Z)$).