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Caramuel's Theory of Opposition

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Abstract

In 1654, the Spanish philosopher and theologian Juan Caramuel y Lobkowitz published his *Theologia rationalis* which contains many interesting observations relevant for the Square of Opposition. In addition to the usual opposition of propositions, Caramuel also investigates the opposition of *terms*, (e.g., Human, Brute, Not brute, Not human). Furthermore, besides the traditional opposition of the *categorical forms* (Every S is P; No S is P; Some S are P; Some S aren't P), Caramuel takes into account the opposition of *modal* propositions (Necessarily q; Impossibly q; Possibly q; Possibly not-q), and of ≫exclusive≪ propositions (Only S are P; Only S are not-P; Not only S are not-P; Not only S are P). Caramuel's most important innovation, however, consists in the development of so-called oblique logic which deals with *doubly quantified* propositions of the type 'Every S' (or Some S, or No S) stands in a certain relation Q to every P (or to some P, or to no P). A detailed analysis of his ≫Cartesian≪ examples concerning all possible types of error (e.g., 'Everyone errs in something', 'No one errs in everything', 'Someone errs in nothing', etc.) is given to show that Caramuel discovered almost all relevant logical relations between these propositions. The relations of opposition and subalternation can be arranged so as to form an octagon (or a cube) of opposition comparable to that of Buridan.

1 Caramuel's Life and Work

Juan Caramuel y Lobkowitz was born in Madrid, Spain, in 1606 and he died in Vigevano, Italy, in 1682. According to Wikipedia, he had a lot of intellectual interests and talents. Already as a child he was occupied with difficult mathematical problems and later on he worked as a Catholic cleric, philosopher, theologian, astronomer, and mathematician. He published at least 60 books, of which, however, "only little was of lasting importance".[1] His *logical* oeuvre

[1]Quoted according to the Wikipedia entry "Juan Caramuel y Lobkowitz", online access on October 17, 2016.

basically consists of two books: *Rationalis et realis philosophia* of 1642 and *Theologia rationalis* published in 1654.[2]

In the standard historiography of logic, Caramuel is either ignored or grossly underestimated. Thus he isn't mentioned at all in Bocheński's *Formale Logik* or in Kneale's *The Development of Logic*, while in Risse's *Die Logik der Neuzeit* he is discredited as "one of the weirdest thinkers whose ideas were full of wit rather than correct".[3] Risse disqualifies Caramuel's logic as "abounding with idiosyncrasies", and he disregards *Theologia rationalis* simply because the title of this book is, admittedly, a bit inappropriate.

It was not before the 20th century that a certain rehabilitation of Caramuel's logical work took place. One of the earliest appreciations was expressed by Pastore (1905) who argued that Caramuel invented a theory of the "Quantification of the Predicate" comparable, if not superior, to that of Hamilton.[4] More recently, Caramuel's logic has been investigated quite extensively by the Czech authors Stanislav Sousedík, Karel Berka, and Petr Dvořák. The main focus of their research lies on Caramuel's sophisticated theory of relational logic.[5]

My reconstruction of Caramuel's theory of opposition is mainly based on *Theologia Rationalis.* This rather voluminous book is subdivided as follows:

[2]Cf. the online-bibliography organized by Jacob Schmutz https://web.archive.org/web/-20070703162930/http://www.ulb.ac.be/philo/scholasticon/bibcaramuel.html.

[3]Cf. Risse (1970), pp. 351–354.

[4]Cf. Pastore (1905), pp. 125–135.

[5]Cf. in particular Dvořák (2006).

My thanks are to the Staatsbibliothek, Berlin, and to the Bayerische Staatsbibliothek, Munich, for providing me with a microfilm and with digital scans of Caramuel's logical works.

2 General Conception of Opposition

Part III of *Grammatica Audax* begins with a "Meditation" on Logic in which the basic categories of terms and propositions are explained. The *opposition* of two propositions is defined to obtain when they "have the same subject and predicate and yet one is affirmative and the other negative". If both propositions are *universal*, then their opposition is called *contrary*; if both are *particular*, their opposition is called *subcontrary*; and if one proposition is universal while the other is particular, their opposition is called *contradictory*. These definitions are illustrated by means of examples as 'Every man is white' vs. 'No man is white' (contrary opposition), 'Some man is white' vs. 'Some man isn't white' (subcontrary opposition), and 'Every man is white' vs. 'Some man isn't white' — and equally 'No man is white' vs. 'Some man is white' — for contradictory opposition. The different types are then characterized by the following semantic conditions:

> Two contrary propositions can, and usually are, together false, but they cant be together true. Two subcontrary propositions usually are [and at any rate can be] together true, but not together false. Two contradictory propositions can neither be together false, nor together true.[6]

The general discussion of opposition is picked up in article 34 of part II of *Logica Vocalis* where the very idea of opposition is explained by saying that the propositions must differ in their ≫quality≪, i.e. in their being either affirmative or negative. The first and most typical opposition obtains between singular propositions with exactly the same subject, e.g. 'Socrates is learned' vs. 'Socrates is not learned'. This paradigmatic example satisfies the condition that the propositions can "neither be together true, nor be together false", provided that the relevant temporal, spatial and other "circumstances" are completely the same. Of course, no real opposition obtains when one and the same individual is sitting at some time in some place but not sitting at another time or in another place.[7]

[6] Cf. *Theologia rationalis* (*TR*, for short), p. 69.

[7] Cf. *TR*, p. 206: "(...) ut patet in his propositionibus *Petrus sedet Praga, Petrus non sedet Roma*, quae oppositionem nullam habent".

In the later course of this article, Caramuel arranges the various forms of opposition in a schema by distinguishing between "adequate" and "inadequate" opposition. *Adequate* opposition is said to obtain when there is a conflict both in regard to *truth* and in regard to *falsity*, i.e. when the propositions are *contradictory* to each other. If they are opposed in only one aspect, i.e. either in regard to falsity (as in the case of *subcontrary* propositions) or in regard to truth (as in the case of *contrary* propositions), the opposition is called *inadequate*.[8]

3 Opposition of Terms

Caramuel defines two *terms* S, T to be *contrary* if and only if no object x can simultaneously have both properties while it is possible that x neither has property S nor property T. Similarly, S and T are *contradictory* if and only if each x either has property S or has property T. Furthermore, S and T are *subcontrary* if and only if each x must have at least one property S or T, while it is possible that x has both properties. Finally, S and T are *disparate* if and only if an object x can have any combination of these properties, i.e. x can be (i) both S and T, (ii) S but not T, (iii) T but not S, or (iv) not S and not T.[9] These definitions are illustrated by means of diagrams which strongly resemble the usual square of opposition. The first example is based on the traditional conception 'Homo est animal' according to which every man is an "animal", i.e. a living being.

(A)	Homo	Non-animal	(E)
(I)	Animal	Non-homo	(O).

Hence the terms (A) and (I) are "subaltern". By contraposition it follows that, whenever x is a "not-animal", then x is a "not-man", i.e. (O) is subaltern to (E). Furthermore (A) and (0) are trivially opposed to each other as *contradictories*, and the same holds for (E) and (I). Moreover (A) and (E) are *contrary* to each other, because (1) if x is a man, then x can't be a not-animal, and (2) there exist some non-human animals, e.g., horses, which neither have property (A) nor property (E). The same example finally shows that the terms (I)

[8] Furthermore Caramuel explains in *TR*, p. 208, that the ≫true≪ or most genuine form of opposition is that between two singular propositions like 'Peter is white' and 'Peter is not white'. This opposition is also called *formally contradictory*. In contrast, the opposition between 'Every man is white' and 'Some man is not white', although being adequate, is not a formal but rather a ≫*radical*≪ contradiction which means that these propositions *logically entail* a pair of formally contradictory singular propositions.

[9] Cf. *TR*, p. 204.

and (O) are *subcontrary.* Hence all logical conditions of a square of opposition are satisfied. In contrast, the subsequent example

(A)	Album	Dulce	(E)
(I)	Non-dulce	Non-album	(O)

lacks most features of such a square. It only illustrates the fact that *disparate* terms like 'White' and 'Sweet' together with their negations 'Not-white' and 'Not-sweet' can, singly or jointly, be present, or absent in certain "objects". As Caramuel remarks: "Sugar is white and sweet, honey is sweet but not white, paper is white but not sweet, [mud] is neither sweet nor white".[10]

4 Opposition of Propositions

In accordance with the terminology of the Scholastics, Caramuel draws a basic distinction between "simple" or *categorical* propositions on the one hand and composite or *hypothetical* propositions on the other hand. While the categorical propositions are further divided, according to their "quantity", into singular, particular, universal, and "indefinite" propositions,[11] the main types of "hypothetical" propositions comprise conjunctions, disjunctions, conditionals, causal propositions, and "exceptive" or "exclusive" propositions.[12] The truth-functional connectives of *conjunction* and *disjunction* will be investigated in section 4.1. In 4.2 the non-truth-functional connectives of *causal* ('because') and *conditional* propositions ('if, then') will briefly be examined. Section 4.3 deals with modal propositions, while 4.4 is devoted to various types of *categorical* propositions. In section 4.5, we briefly consider *exceptive* and *exclusive* propositions before we turn to *relational* propositions in 4.6.[13]

[10]Caramuel's example 'snow' ("nix") has been replaced by 'mud'. Cf. *TR*, p. 205: Saccharum enim est album & dulce; mel dulce & non album, charta alba & non dulcis, nix nec dulcis nec alba.

[11]An "indefinite" proposition, however, doesn't constitute a genuine category besides universal and particular proposition, but rather is taken either as a universal or as a particular proposition.

[12]Cf. *TR*, p. 68, and p. 72: "Hypothetica dividitur in Copulativam (&), Disjunctivam (*vel*), Causalem (*quia*), Reduplicativam (*pro ut*), Conditionalem (*si*)".

[13]In addition, as a special case of relational propositions one might consider propositions with a "quantified predicate", but these have already been investigated in Lenzen (2015), pp. 361–384.

4.1 Truth-functional propositions

The traditional definition according to which contradictory propositions "can neither be together false, nor together true" basically captures the logical feature of the negation operator as it is nowadays formalized by the truth-table:

p	T	F
$\neg p$	F	T

This conception immediately entails that the negation of the negation of p has the same truth-conditions as p, i.e. both propositions are logically equivalent. Strangely enough, Caramuel nowhere *explicitly* put forward this *law of double negation* although he occasionally made use of it in an implicit way. On the one hand, when elaborating his theory of contraposition, he replaced a doubly negated *term* like 'non non lapis' by the simple term 'lapis'.[14] On the other hand, during his discussion of the inference schemata of *modus ponens* and *modus tollens*, he repeatedly replaced a doubly negated *proposition* like 'Rex non non vivit' by the simple proposition 'Rex vivit'.[15]

In article 6 of the second part of *Logica Vocalis*, Caramel deals with the *conjunction* of two propositions. Unfortunately, he mainly focuses on grammatical rather than logical distinctions. Thus he points out that a conjunction with a *single* subject and a *double* predicate such as 'Peter is sitting and eating' can equivalently be transformed into the conjunction of *two* propositions with a *single* predicate, Peter is sitting and Peter is eating. The systematically more important fact, that the truth of such a conjunction requires that both components be true, is only mentioned incidentally. Similarly, Caramuel explains at some length that a conjunction with *two subjects* such as 'Peter and Paul are walking' can be transformed into a conjunction of two propositions with a *single* subject 'Peter is walking and Paul is walking'. Only afterwards he adds the decisive semantic condition that it is necessary "that both are true; hence, if the one or the other is not walking, the conjunctive proposition is entirely false". Finally, however, Caramuel states the general truth-conditions for a conjunctive proposition in the following abstract way:

> It makes two assertions, and for its truth it would not be enough that one of them is true, but it is necessary that both are true.[16]

[14]Cf. *TR*, p. 73: "(...) omne non-non-lapis est non homo, Ergo omnis lapis est non homo".

[15]Cf. *TR*, p. 483: "*Si Princeps coronaretur, Rex non viveret. At Rex non non vivit* [vel clarius, *At Rex vivit*], *Ergo Princeps non coronatur*".

[16]Cf. *TR*, p. 171: "Duo quidem enunciat, nec sufficeret quod alterutrum esset verum, ut ipsa esset vera, sed est opus ut sit verum utrumque".

This requirement is basically equivalent to the modern truth-table:

p	q	$p^{\wedge}$
T	T	T
T	F	F
F	T	F
F	F	F

The subsequent article 7 deals with *disjunctive* propositions as they are expressed by means of the particle 'vel' ('or'). Their truth-conditions would nowadays be explicated by the table:

p	q	$p^{\vee}$
T	T	T
T	F	T
F	T	T
F	F	F

In a somewhat less formal way, Caramuel notes that a disjunction combines *two* assertions:

> But since they are asserted in a disjunctive way, they will be true if one of them is true; they will be false, if none of them is true.[17]

This formulation leaves open, however, whether in the case where *both* propositions are true, the disjunction shall be considered as true or false. Usually logicians distinguish between an *exclusive* interpretation ('aut') and a *non-exclusive* interpretation ('vel') of disjunction, but apparently no such distinction was made by Caramuel. In a brief "Disputation" of disjunctive propositions, he only pointed out that their "*conversion*" is unproblematic, i.e. the two members of a disjunction can always be interchanged.[18] But this feature of symmetry holds no matter whether disjunction is taken in the exclusive or in the non-exclusive sense.

Unfortunately, Caramuel abstained from a closer investigation of the *negation* of disjunctive propositions. He didn't want to subscribe to what others have written about this "dangerous" issue and contented himself with noting that such a negation can at any rate be obtained by just putting 'not' (or 'It

[17] Cf. *TR*, p. 172.

[18] Cf. *TR*, p. 418: "De Disiunctivarum Conversione multa non dicam, est enim facillima & tutissima ut patet in datis exemplis (...) *Petrus est albus, vel Petrus est niger: Ergo Petrus est niger, vel Petrus est albus.*"

is false that') in front of the entire proposition.[19] Apparently Caramuel never came even close to formulating the so-called De Morgan laws concerning the negation of conjunctive and disjunctive propositions, although these laws were familiar already to medieval logicians like W. Ockham.[20]

4.2 Conditionals and causal propositions

In contrast to the modern interpretation of 'If p, then q' as a *material implication*, Caramuel's understanding of conditionals is not (fully) truth-functional but rather *causal*. The antecedent, p, is supposed to be the reason why if p is taken to be true, also the consequent q has to be taken to be true.[21] Typical examples of true conditionals are 'If Peter is running, then he moves' and 'If the sun is appearing, it is day'. The difference between a *conditional* 'If p, then q' and a *causal* proposition 'q, because p' basically consists in that the latter implicitly affirms the truth of the antecedent (and therefore also of the consequent). E.g., 'Peter is moving because he is running' is taken by Caramuel to mean 'Peter is running and he is in fact moving, and he is moving because he is running'. In contrast, the mere conditional 'If Peter is running, he is moving' roughly means:

> I do not decide whether he is running, for I do not know that, or I don't want to argue that, yet I say that from the assumption that he is running [it follows that] he is moving.[22]

Although Caramuel dealt with conditionals in several parts of his *TR*,[23] he doesn't have much to say about their logic. On the one hand, he only notes that conditionals cannot be converted, i.e. 'If p, then q' doesn't generally entail 'If q, then p'.[24] On the other hand, he simply points out that one may safely form the negation of a conditional by putting 'not' (or 'it is not true that') in

[19] Cf. *TR*, p. 418: "Nec multa subscribam de earundem oppositione nam quidquid alii uberius & periculosius ingeminent, ego sic quaestionem expedio."

[20] Cf. W. v. Ockham *Summa Logicae*, (ed. by Ph. Boehner, G. Gál & St. Brown), St. Bonaventure, N.Y., 1974, Part II, p. 348: "Sciendum est etiam quod opposita contradictorie copulativae est una disiunctiva composita ex contradictoriis partium copulativae". Similarly, p. 350: "Sciendum est etiam quod opposita contradictorie disiunctivae est una copulativa composita ex contradictoriis partium ipsius disiunctivae".

[21] Cf. *TR*, p. 418: "(...) unde colligi omnem conditionalem duas dicere propositiones (antecedentem & consequentem) & illam priorem esse causam (intrinsecam, extrinsecam, aut illativam) qua posita etiam poni debeat posterior."

[22] Cf. *TR*, p. 419.

[23] Cf. article 15 of part 2 of *Logica Vocalis* (*TR*, p. 179–183) and "Disputatio IV" of part 1 of the section on oblique propositions (*TR*, p. 418–420).

[24] Cf. *TR*, p. 419: "Conversio conditionalium periculosissima est, rarò enim veritas conservatur".

front of the entire proposition. However, instead of 'It is not true that Peter moves if he is running' one can also say 'Peter doesn't move if he is running', and 'It is not true that it is day if the sun is appearing' may similarly be simplified into 'It is not day, if the sun is appearing'. Furthermore, according to Caramuel, the negation of a *subjunctive* conditional can be formed by negating the consequent. E.g., 'If a lion would be a man, it would be rational' can be negated in this way: 'If a lion would be a man, it would not be rational'.[25]

4.3 Modal propositions

The following figure displays the logical relations between the (alethic) modal operators:

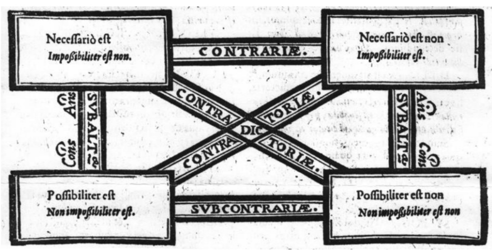

Figure 1: Square of Modal Propositions (scanned from *TR*, p. 73)

This diagram shows that Caramuel was fully aware of the fact that the alethic modalities can be defined in terms of just one modal operator (plus suitably chosen negations). Thus, e.g., 'Necessarily p' can be defined as 'Impossibly $\neg p$'; 'Possibly p' as 'Not impossibly p'; and 'Possibly $\neg p$' as 'Not impossibly $\neg p$'. Now in a separate investigation of the problem of contingency, Caramuel further draws an interesting parallel between modal and categorical propositions:

> NECESSARY is equivalent to the Universal Affirmative, since it entails universal consequences, for when we say, *Is it necessary? It will therefore always be [the case], or everywhere, or at any time, or in every place, or on every occasion.* (...)
>
> IMPOSSIBLE is equivalent to the Universal Negative, since it entails universally negative consequences, as when we say *Is it impossible? It will therefore never be or have been [the case], or nowhere, or at no time, or in no place, or on no occasion* &tc.
>
> POSSIBLE is subaltern to Necessary, for the inferences *It is necessary, therefore it will also be possible or It is not possible, therefore*

[25] Cf. *TR*, p. 419.

> *it is not necessary* are valid. (...)
>
> Thus it must be said that *Possibly* and *Possibly not* are equivalent to particular propositions, though not concerning the meaning, but concerning the opposition, for they are opposed in the same way as *Something* and *Something not* (...).[26]

If the expression '*on every occasion*' ("in omni occasione") is interpreted as 'in all *possible cases*', then Caramuel's analysis may be considered as an early version of *possible-worlds-semantics.*

4.4 Categorical propositions

As was already mentioned in section 2, Caramuel fully subscribes to the traditional doctrine according to which the UA and the UN are opposed as *contraries*, while the particular affirmative (PA) and the particular negative (PN) proposition are opposed as *subcontraries.* Furthermore the opposition between the UA and PN — and similarly between the UN and the PA — is that of a *contradiction.* In view of the principle of *subalternation*, which Caramuel defines as the "descent from a universal to a particular proposition",[27] one obtains the well-known square of opposition:

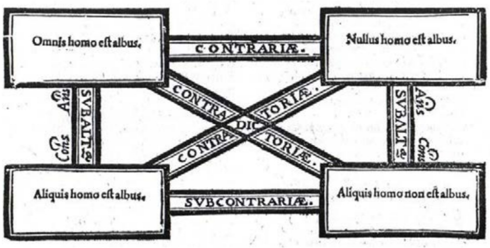

Figure 2: Square of Oppositions (scanned from *TR*, p. 69)

Caramuel points out that the terminological distinction between contrary and contradictory opposition goes back to Aristotle, and he argues that the traditional doctrine should be slightly modified. A contradictory opposition obtains not only between *universal* and *particular* propositions, but rather between *universal* and *non-universal* propositions where the latter also cover *singular* propositions. Thus 'Every man is coloured' is not only opposed to 'Some man is

[26] Cf Caramuel (1655), p. 36, "Articulus I *De Oppositione Modalium*".

[27] Cf. *TR*, p. 69, where Caramuel not only considers the usual subalternation of the *subject*, i.e. the inference from, e.g., 'Omnis homo est animal' to 'Ergo aliquis homo est animal', but also an unorthodox subalternation of the *predicate*, i.e. the inference "Aliquod animal est omnis homo. Ergo aliquod animal est aliquis homo."

not-coloured' but also to 'Peter is not-coloured'. Caramuel thinks that Petrus Hispanus' theory of the opposition of singular propositions has to be corrected accordingly.[28] But Caramuel himself commits a minor mistake when he characterizes the opposition between the singular and the universal proposition as *contradictory*, while in fact it is *contrary*.[29]

More interestingly, Caramuel extends the usual square to the following figure where each categorical proposition is formulated in three different ways:

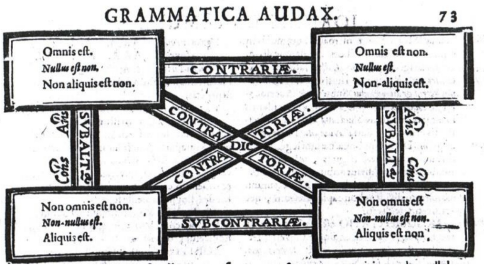

Figure 3: Extended Square of Oppositions (scanned from *TR*, p. 73)

Hence all three informal quantifier expressions 'Every', 'Some' and 'No' can be defined in terms of each other.

Moreover Caramuel points out that the ordinary laws of opposition no longer hold if the categorical propositions are taken in the *plural sense* so that 'Some S are P' means that there are *several* things which are both S and P. In this case, two opposite universal propositions as, e.g., 'All men are white' and 'No men are white' remain contrary to each other. But the particular propositions 'Some men are white' and 'Some men are not white' are no longer subaltern because they might both be false. E.g., if there exist just two men, one of them being white, the other not-white, then neither the PA nor the PN are true (in the plural sense). Accordingly, the opposition between the UA and the PN, and similarly between the UN and the PA, no longer is contradictory but rather contrary because in the aforementioned case all propositions would be false. Therefore also the laws of subalternation fail to hold in this case.[30]

[28] Cf. *TR*, p. 207: "Unde colligo omnem [oppo]sitionem inter universalem & particularem (modò una neget quod alia asserit) contradictoriam esse; non autem omnem contradictionem his occludi limitibus, quia & singularis singulari, & universali opponitur contradictoriè. Hinc venit corrigendus Petrus Hispanus *lib.* 1, cap. 11 qui intermisit oppositionem singularium."

[29] To be sure, the truth of 'Peter is not coloured' entails the falsity of 'Every man is coloured', but conversely the falsity of 'Every man is coloured' does not entail that *Peter* is not-coloured; only *some* man (either Peter, or Paul, or someone else) must be not-coloured!

[30] Cf. *TR*, p. 211-3.

4.5 "Exclusive" and exceptive propositions

In Caramuel's opinion, "exclusive" propositions like 'Only just people are happy' ("Solus justus est felix") consist of two clauses. The "exclusion" clause always has to be understood in a universal sense. In our example it says that whoever is not just, can't be happy. The "inclusion" clause, however, can be understood either as universal or as particular. In the given example it would affirm either that *all* just people or that *at least some* just people are in fact happy.[31]

More generally, according to Caramuel, a universal "exclusive" proposition maintains that all F are G, but no one else is G. With the help of modern predicate logic, this condition can be formalized as follows:

(UEx) $$\forall x(Fx \rightarrow Gx) \wedge \forall x(\neg Fx \rightarrow \neg Gx).$$

The *particular* variant instead maintains that some F are G, but no one else is G, i.e.:

(PEx) $$\exists x(Fx \wedge Gx) \wedge \forall x(\neg Fx \rightarrow \neg Gx).$$

Caramuel briefly considers negated versions of "exclusive" propositions such as 'Not only men are living beings'. He believes that the negation particle here only affects the "excluded" clause, so that the proposition becomes equivalent to 'Men are living beings but there are other living beings besides men'. However, no matter whether 'Men are living beings' is taken in the universal or in the particular sense, the entire conjunction certainly is not the *negation*, i.e. the *contradictory* opposite, of 'Only men are living beings'.

More generally, if 'Only F are G' is assumed to contain an "inclusion" clause like $\forall x(Fx \rightarrow Gx)$ or $\exists x(Fx \wedge Gx)$ as a *conjunct*, then its negation must be conceived of as a *disjunction* of two propositions, one of which is the negation of the "exclusion" clause $\forall x(\neg Fx \rightarrow \neg Gx)$, while the other is the negation of the "inclusion" clause, i.e. either the negation of $\forall x(Fx \rightarrow Gx)$ or the negation of $\exists x(Fx \wedge Gx)$. This point appears to have been recognized already by Scholastic logicians, who considered, e.g., 'Non tantum homo est animal' as equivalent to 'Vel nullus homo est animal, vel aliquid aliud ab homine est animal'. But Caramuel thinks they were mistaken because according to their analysis the related example 'Non tantum homo est lapis' (which in his opinion is clearly false) would have to be considered as *true* because it might be transformed into 'Vel nullus homo est lapis, vel aliquid aliud ab homine est lapis'.[32]

[31] Cf. *TR*, p. 172-3. Occasionally Caramuel also considers the indefinite version where it remains open whether it is to be understood in the universal or in the particular sense.

[32] Cf. *TR*, p. 173: "Sed videant Eruditi (ait Hunnaeus, quem contra Dialecticos seniores

The issue of the negation of ≫exclusive≪ propositions is taken up in article 37 of part 2 of *Logica Vocalis* where Caramuel considers the example 'Only men are white' ("Tantum homo est albus"). By inserting one or two negation particles 'non', he obtains the following diagram:

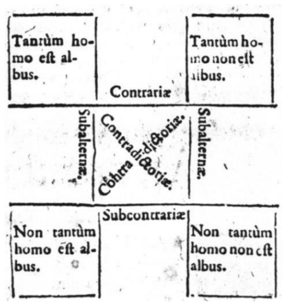

Figure 4: Square of "exclusive" propositions (scanned from *TR*, p. 216)

However, in order to satisfy the usual requirement that the diagonally opposed elements negate each other, the two propositions at the bottom of this diagram evidently have to be interchanged! But even after the due correction:

(A)	(A) Only men are white	Only men are not white	(E)
(I)	Not only men are not white	Not only men are white	(O).

it remains unclear whether the figure really forms a *square of opposition.* At least it is far from evident whether (I) and (O) are *subcontrary* propositions and whether the *subalternation* from (A) to (I) and from (E) to (O) do hold. In order to clarify this issue, let us consider another diagram which also has the shape of a *square* but which was not meant as a square of *opposition.*

sequor) annon secundum usitatam Latiné loquentium consuetudinem haec sit falsissima, *Non tantum homo est lapis.* Cuius tamen expositio, si veterem exponendi modum sequamur, erit vera: sic enim sequendo illum modum exponi deberet; *Vel nullus homo est lapis, vel aliquid aliud ab homine est lapis*".

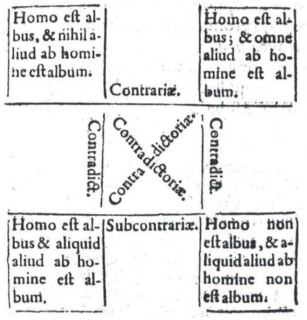

Figure 5: Opposition among ≫exclusive≪ propositions (scanned from *TR*, p. 217)

This diagram contains several mistakes. Given that for Caramuel an ≫exclusive≪ proposition always contains a corresponding ≫inclusion≪-clause, the (A)-proposition 'Homo est albus & nihil aliud ab hominem est album' may be viewed as a paraphrase of the previous 'Tantum homo est albus'. This does not, however, hold for the (E)-proposition 'Tantum homo non est albus' and its attempted paraphrase 'Homo est albus & omne aliud ab homine est album'. Here the ≫inclusion≪-clause evidently has to be corrected to 'Homo non est albus'!

Furthermore, Caramuel's claim that the subcontrary propositions (I) and (O) would *both* be "contradictories" of (A) is plainly false.[33] In general, if one has three propositions α, β, and γ such that β is the negation of α and also γ is the negation of α, it necessarily follows that β and γ are logically equivalent. However, 'Homo est albus & aliquid aliud ab homine est albus' is certainly not logically equivalent to 'Homo non est albus & aliquid alius ab homine non est album'.

Fortunately, Caramuel's schema may easily be improved so as to yield a correct square of opposition. If one simply drops the "inclusion"-clauses 'Homo est albus' and 'Homo non est albus', and if the (I)- and (O)-propositions are interchanged, one obtains:

Nihil aliud ab homine est album	Omne aliud ab homine est album
Aliquid aliud ab homine non est album	Aliquid aliud ab homine est album

This result fully accords with the analysis of the logical relations between *"exceptive"* propositions which Caramuel summarizes in a short paragraph "De oppositione Exceptivarum" (*TR*, p. 217):

[33]This claim not only follows from the corresponding "arrows" of the diagram but it was also emphasized in the text: "(...) illa propositio *Tantum homo est albus* utrique subcontrariae contradicit" (*TR*, p. 217).

> (...) to this proposition *Every animal besides man is white* the following is opposed as a contrary: *No animal besides man is white.* And there is a subalternation from these contrary propositions to the subcontrary propositions *Some animal besides man is white*, *Some animal besides man is not white.*[34]

More generally, it seems advisable to interpret "exclusive" propositions of the form 'Only F are G' not as entailing that all or at least some F are in fact G, but rather as merely asserting that if something is *not* F, it can't be G either:

(Ex) $\quad \forall x(\neg Fx \rightarrow \neg Gx)$, i.e. $\forall x(Gx \rightarrow Fx)$.

Under this interpretation the square of opposition for *"exclusive"* (or, for that matter, also of *"exceptive"*) propositions eventually turns out to be just a variant of the square for *categorical* propositions:

(A)	Only F are G $\forall x(Gx \rightarrow Fx)$	Only not-F are G $\forall x(Gx \rightarrow \neg Fx)$	(E)
(I)	Not only not-F are G $\exists x(Gx \wedge Fx)$	Not only F are G $\exists x(Gx \wedge \neg Fx)$.	(O)

4.6 Relational propositions

Caramuel's theory of "oblique" propositions is mainly developed in sections 10-14 of part 1 of *Logica Obliqua* (*TR*, pp. 410-4). Section 10 starts with a survey of the various kinds of affirmative and negative propositions with a binary predicate $F(a, b)$ such as:

(UA)	Omnes homines vidit Petrus	$\forall x(Hx \rightarrow F(a, x))$
(UN)	Nullos homines vidit Petrus	$\forall x(Hx \rightarrow \neg F(a, x))$
(PA)	Aliquem hominem vidit Petrus	$\exists x(Hx \wedge F(a, x))$
(P)	Aliquem hominem non vidit Petrus	$\exists x(Hx \wedge \neg F(a, x))$
(N)	Ioannem vidit Petrus	$F(a, b)$
(SN)	Ioannem non vidit Petrus	$\neg F(a, b))$.[35]

[34] In general Caramuel sees no big logical difference between ≫exclusive≪ and ≫exceptive≪ propositions. As he explains, e.g., *TR* p. 175, 'Nullus praeter hominem est risibilis' can be transformed into 'Solus homo est risibilis'. Further discussions of ≫exclusive≪ and ≫exceptive≪ syllogisms may be found in the 5$^{\text{th}}$ part of *Logica Vocalis*; cf. *TR*, pp. 340-4.

[35] Besides these *singular* propositions Caramuel also considers *indefinite* propositions ("Hominem vidit Petrus"; "Hominem non vidit Petrus") and propositions containing *definite descriptions* ("Hunc hominem vidit Petrus", "Hunc hominem non vidit Petrus"). These, however, may be ignored in what follows.

Next Caramuel points out that the two propositions ‘Non vidi aliquem Hispanum Roma’ and ‘Roma aliquem Hispanum non vidi’ are not equivalent to each other. This fact might as well be illustrated by means of propositions from the above stock of examples:

Non aliquem hominem vidit Petrus	$\neg\exists x(Hx \wedge F(a,x))$
Aliquem hominem non vidit Petrus	$\exists x(Hx \wedge \neg F(a,x))$

Caramuel explains this difference by the rule: “(...) if the negation precedes the syncategorematic expression ‘some, it doesn’t affect the verb but [the quantifier]; if the negation follows [the quantifier], it is the other way round”.

4.6.1 Caramuel’s Symbolism for the Quantity of Propositions

Caramuel mentions the traditional symbols ‘A’, ‘E’, ‘I’ and ‘O’ for the quality and quantity of propositions and explains that he will refrain from using ‘O’ and take instead ‘N’ as a symbol for *negation*. More exactly he uses two different negation symbols ‚N‘ and ‚Ñ‘ (sometimes written as ‚n‘ or ‚ñ‘) depending on whether it refers to the subsequent *quantifier* (‘N’/‘n’) or to the *copula* (Ñ/ñ). This distinction, however, is systematically unimportant and it was also abandoned by Caramuel himself when he later developed his theory of “oblique syllogisms” in “Disputations” 11-13.[36]

By way of combining ‘A’, ‘E’, and ‘I’ with a negation symbol, Caramuel obtains nine different expressions, viz. A, E, I, AN, EN, IN, NA, NE, and NI.[37] From a systematic point of view, however, it would be better to adduce also doubly negated expressions NAN, NEN, and NIN, so that the entire set of complex symbols can be arranged as follows:

A	$\leftrightarrow$	EN	$\leftrightarrow$	NIN
AN	$\leftrightarrow$	E	$\leftrightarrow$	NI
NA	$\leftrightarrow$	NEN	$\leftrightarrow$	IN[38]
NAN	$\leftrightarrow$	NE	$\leftrightarrow$	I.[39]

[36] Cf. *TR*, p. 442–457; “Disputatio XII”, p. 458–464, and “Disputatio XIII”, p. 464–467.

[37] Cf. *TR*, p. 411, left column. As a matter of fact, he mentions also the sequence I*IN which, however, belongs to a quite different context.

[38] This line, of course, contains various expressions equivalent to the traditional ‘O’!

[39] The topic of equivalent expressions is further treated in article 11 (“De aequipollentia Propositionum”). Caramuel begins with the following list which is neither very systematic nor complete: A $\leftrightarrow$ EN; E $\leftrightarrow$ AN; I $\leftrightarrow$ NE; NI $\leftrightarrow$ E; and IN $\leftrightarrow$ NEN. Towards the end of

Next Caramuel explains that in an oblique proposition both the subject containing the connotating term and the predicate containing the connotated term may be affected by a quantifier. E.g., in Nullus homo videt aliquem angelum the connotating term homo is modified by omne while the connotated term angelus is modified by aliquis.

4.6.2 Caramuel's Symbolism for Propositions with two Quantifiers

In section 11 Caramuel develops the following formalism for oblique propositions with two quantifiers where the symbol for the connotating term is separated from the symbol for the connotated term by an asterix '*'. The first group contains various equivalent formulations of a proposition where both terms have quality 'A' (or some equivalent quality as 'EN' or 'NI'):

A*A	Omnis sanctus amat omnem proximum
E*NA	Nullus sanctus non amat omnem proximum
E*IN	Nullus sanctus aliquem proximum non amat
A*EN	Omnis sanctus nullum proximum non amat.

In the second group the first term has quality A while the second term has quality E:

A*E	Omnis sanctus habet nullum peccatum
E*I	Nullus sanctus habet aliquod peccatum
NI*I	Non aliquis sanctus habet ullum peccatum.

In the third group the first term has quality A and the second term quality I:

A*I	Omnis homo habet aliquem defectum
E*E	Nullus homo habet nullum defectum
NI*E	Non aliquis [homo] habet nullum defectum
NI*NI	Non aliquis [homo] non habet aliquem defectum.

the article the most important equivalences are once again summarized as follows: A $\leftrightarrow$ EN; E $\leftrightarrow$ AN $\leftrightarrow$ NI; I $\leftrightarrow$ NE; IN $\leftrightarrow$ NE $\leftrightarrow$ NA. Note, incidentally, that Caramuel accepts the equivalence between NA and IN only provisionally. In the second part of "Herculi Logici Labores Tres" he explains at greater length that 'Non omnis homo currit' is not just the negation of 'Omnis homo currit' but rather means 'Aliquis homo currit' & 'Aliquis homo non currit'.

In the fourth group the first term has quality A and the second term quality IN (i.e., O):

A*IN	Omnis monoculus aliquem oculum non habet
A*NEN	Omnis monoculus nonnullum oculum non habet
E*A	Nullus monoculus habet omnes oculos
A*NA	Omnis monoculus habet non omnes oculos.

The result of these considerations is then summarized in the following table which was praised by Julian Velarde as a "precise and complete symbolism which Caramuel invented for the exposition of the relations and operations between relational propositions":[40]

(EQ 1)	A*A	E*NA	E*IN	A*EN
(EQ 2)	A*E	E*I	NI*I	
(EQ 3)	A*I	E*E	NI*E	NI*NI
(EQ 4)	A*IN	A*NEN	E*A	A*NA
(EQ 5)	E*A	A*IN	A*NEN	A*NA
(EQ 6)	E*E	A*I	NI*E	NI*NI
(EQ 7)	E*I	A*E	NI*I	
(EQ 8)	E*IN	A*A	E*NA	A*EN.[41]

As a matter of fact, however, Caramuel's table is somewhat incomplete in so far as it misses the combinations I*I, I*E, I*I, and I*IN. Furthermore, it is quite redundant since the equivalences listed in EQ 1 are exactly the same as those in EQ 8 (only in a different order); and the same holds for EQ 2 and EQ 7; EQ 3 and EQ 6; and EQ 4 and EQ 5.

4.6.3 Caramuel's Theory of Opposition for Oblique Propositions

At the beginning of section 12 the reader is referred to a sheet with examples of oblique propositions and their negations.[42] The ninth figure contains the

[40] Cf. Velarde (1984), p. 275.

[41] Cf. *TR*, p. 411, right column; the "names" of the equivalences, of course, are not Caramuel's but ours. The original table differs from my reconstruction in some respects. In particular, it has 'N' and 'Ñ' for negation, and the symbols are printed sometimes in small and sometimes in capitals letters. Thus the last line of Caramuel's table originally reads: "E*(i)ñ. A*a. E*ña. A*(e)ñ."

[42] Cf. *TR*, p. 412: "Laminam secundam adito & in ea figuram IX, X, XIII, XIV, XVII, & XVIII contemplator". Such an extra leaf was not, however, contained in my copy of *TR*.

propositions 'Video omnes', 'Video nullos', 'Video aliquos', 'Aliquos non video', which are transformed into the passive so that the quantifiers uniformly stand at the beginning of the sentence:

(A)	Omnis videtur à me	Nullus videtur à me	(E)
(I)	Aliquis videtur à me	Aliquis non videtur à me	(O)

Caramuel only remarks that the question of opposition here makes no problem.[43] Next he turns to various propositions containing two quantifiers each. The meaning of the following group of examples (from the tenth figure) "Sum omnia omnibus", "Sum nulla omnibus", "Sunt aliqua qua sum omnibus", "Sunt aliqua qua non sum omnibus" remains quite unclear and Caramuel apparently considers all four propositions as false. A similar remark applies to the following variant (figure 17): "Sum omnibus omnia", "Sum nullis omnia", "Aliquibus sum omnia"; and "Aliquibus non sum omnia". Next follows a variant (figure 14) which differs from the previous one only by having plural form instead of singular: "Sum omni omnia"; "Sum nulli omnia"; "Alicui sum omnia"; and "Alicui non sum omnia". A possible interpretation of these sentences is given by the following "paraphrase":

Omnis homo habet in me omnia	Nullus homo habet in me omnia
Aliquis homo habet in me omnia	Aliquis homo non habet in me omnia.

Without going into details, Caramuel says that the opposition and subalternation of these propositions is unproblematic. Next he considers the following group (figure eighteen): "Sum omni omne", "Sum omni nihil", "Sum omni aliquid", and "Est aliquid quod non sum omni". Caramuel suspends the judgment about the subalternation and opposition of these propositions, but he thinks that they may be paraphrased by the following sentences which obey the laws of ordinary logic ("legibus communis Logicae"):

(A)	Omne auxilium datur à me omni amico	Nullum auxilium datur à me omni amico	(E)
(I)	Aliquod auxilium datur à me omni amico	Aliquod auxilium non datur à me omni amico	(O)

[43]Cf. *TR*, p. 412, right column: "De quarum oppositione nulla nova insurgere potest difficultas."

4.6.4 Caramuel's Octagon of Opposition

After these meager results, Caramuel turns to another group of propositions (section 13) whose truth-conditions are much easier to understand. Prompted by a booklet of a certain Cyriacus Lentulus[44] he considers the question whether some, all, or no people can err in some, all, or no affair. In order to facilitate the discussion of Caramuel's densely formulated thoughts, let us first give a survey of all possibilities of doubly quantifying the predicate $E(x, y)$ ('x errs (or is mistaken) in y'). With the help of the modern quantifiers the four affirmative combinations can be displayed as follows:

(DQ 1)	$\forall x \forall y E(x, y)$	(Everyone errs in everything)
(DQ 2)	$\exists x \forall y E(x, y)$	(Someone errs in everything)
(DQ 3)	$\forall x \exists y E(x, y)$	(Everyone errs in something)
(DQ 4)	$\exists x \exists y E(x, y)$	(Someone errs in something).

The negation of these propositions yields the following sequence of formulas:

(DQ 5)	$\neg \forall x \forall y E(x, y)$, or	$\exists x \neg \forall y E(x, y)$, or	$\exists x \exists y \neg E(x, y)$
(DQ 6)	$\neg \exists x \forall y E(x, y)$, or	$\forall x \neg \forall y E(x, y)$, or	$\forall x \exists y \neg E(x, y)$
(DQ 7)	$\neg \forall x \exists y E(x, y)$, or	$\exists x \neg \exists y E(x, y)$, or	$\exists x \forall y \neg E(x, y)$
(DQ 8)	$\neg \exists x \exists y E(x, y)$, or	$\forall x \neg \exists y E(x, y)$, or	$\forall x \forall y \neg E(x, y)$.

"Colloquial" counterparts of these formulas will be discussed in due course. In particular, it remains to be seen how these formulas are related to Caramuel's phrases containing the expression 'nullus' or 'in nullo'.

In a marginal note to §MXLV Caramuel announces a complete logical examination of these propositions (purportedly listed in a "tertia figura" of an inserted "lamina prima"),[45] but in the subsequent text he dismisses the issue of *subalternation* because, allegedly, it needs no explanation. Instead he focuses on the issue of *opposition* and explains:

> Haec propositio *Omnis errat in omni* contrariatur huic, *Nullus errat in omni*, confalsae enim sunt, & esse converae non possunt. Etiam contrariatur huic: *Nullus errat in aliquo*: est enim falsa utraque, nec dabis in simili forma simul versa. At contradicit huic: *Aliquis*

[44] Cf. Cyriacus Lentulus, "Nova Renati des cartes sapientia, faciliori, quam ante hac, methodo detectata", *Herbornae Nassoviorum* 1651.

[45] Again, such an inserted leaf which the reader is referred to ("quaere laminam primam, & considera figuram tertiam") was not contained in my copy of *TR*.

> *non errat in omni*, ac proinde etiam huic: *Aliquis non errat in aliquo*[.]

This passage contains four assertions. (1) 'Everyone errs in everything', i.e. DQ 1, is maintained to be *contrary* to 'No one errs in everything'. The latter proposition has to be formalized as $\neg\exists x\forall yE(x,y)$ and thus it is equivalent to DQ 6, i.e. the negation of DQ 2. Since DQ 1 logically entails DQ 2, DQ 6 must be false if DQ 1 is true, i.e. the two propositions are logically incompatible. Furthermore, if DQ 2 is the case without also DQ 1 being the case, i.e. if *someone* but *not everyone* is mistaken in everything, then DQ 1 and DQ 6 are both false. Hence these propositions are indeed contrary to each other.

(2) DQ 1 is maintained to be contrary also to 'No one errs in something'. The latter proposition has the logical structure $\neg\exists x\exists yE(x,y)$ and thus is equivalent to DQ 8, i.e. the negation of DQ 4. Again, since DQ 1 logically entails DQ 4, DQ 8 must be false if DQ 1 is true, i.e. the two propositions are logically incompatible. Furthermore it is quite imaginable that someone, a, errs in everything while someone else, b, does not err in everything. In this case DQ 1 and DQ 8 would be together false. Altogether, then, DQ 1 and DQ 8 are indeed contrary to each other.

(3) Caramuel rightly remarks that 'Someone does not err in everything', i.e. DQ 5, is the *contradictory* opposite of DQ 1.

But (4) he further maintains that also 'Someone doesn't err in something' would be contradictorily opposed to DQ 1. At first sight this sounds very unlikely because if two propositions β and γ are negations of one and the same proposition α, it logically follows that β and γ must be logically equivalent. However, 'Someone does not err in *everything*' ("Aliquis non errat in *omni*") does not appear to be *equivalent* to 'Someone doesn't err in *something*' ("Aliquis non errat in *aliquo*")! To resolve this difficulty, observe that there is a certain ambiguity in the formulation of DQ 5. As the first formula $\exists x\neg\forall yE(x,y)$ makes clear, the negation here refers to the whole clause 'err in everything' so that it would better be presented by means of parentheses as 'Someone does not (err in everything)'. Now to maintain of someone, a, that it is not the case that a errs in everything is tantamount to maintaining that there exists at least one y such that a does not err in y: $\exists y\neg E(a,y)$. Hence DQ 5 can equivalently be formalized by the second condition $\exists x\exists y\neg E(x,y)$. So if Caramuel's 'Someone does not err in something' ("Aliquis non errat in aliquo") is interpreted as 'Someone in something doesn't err' ("Aliquis in aliquo non errat"), then this proposition does indeed represent an alternative negation of DQ 1.

Next Caramuel goes on to explain:

> Et hinc patet quo opponatur modo haec *Nullus errat in omni* caeteris affirmativis. Disparate sunt *Omnis errat in aliquo*, *Aliquis*

> *errat in omni.* Non repugnat esse simul veras, aut simul falsas, & si agamus de tali hominum genere, & brevi tempore iam erunt confalsae de facto, & iam converae.

At the beginning of this passage, Caramuel maintains that the oppositions between 'No one errs in everything', i.e. DQ 6, and the other affirmative propositions would be evident. Unfortunately, he doesn't give any further hint, so that one has to speculate which oppositions he may have had in mind. However, DQ 6 was already shown above to be *contrary* to DQ 1. Furthermore, DQ 2 is the literal negation of DQ 6. So only the relations between DQ 6 and DQ 3 and between DQ 6 and DQ 4 remain to be illuminated.

In the subsequent sentence Caramuel states that 'Everyone errs in something', i.e. DQ 3, and 'Someone errs in everything', i.e. DQ 2, are *disparate* propositions, which means that the truth of the one is independent from the truth of the other. He argues that nothing prevents them from being together false (if a particular group of men and a particular span of time is taken into account); also nothing prevents them from being together true (if another group of men and another span of time is taken into account). Next he points out:

> Idem dixero de his negativis *Nullum errat in aliquo*[,] *Aliquis non errat in omni* [.] Essent enim converae, si nemo erraret: & confalsae, si omnis erraret in omni.

As was argued above, 'No one errs in something' has to be analyzed as DQ 8. Similarly, the logical structure of 'Someone doesn't err in everything' will naturally be assumed to be $\exists x \neg \forall y E(x, y)$, i.e. DQ 5. As Caramuel pointed out, both propositions would be true in the best of all possible worlds where nobody ever errs. Similarly, both would be false in the worst of all possible worlds where everybody always errs. Yet it is not entirely correct to consider DQ 5 and DQ 8 as *disparate* propositions. Their truth-values are not entirely independent of each other. Since DQ 1 logically entails DQ 4, the negation of the latter, i.e. DQ 8, logically entails the negation of the former, i.e. DQ 5. In other words, there is a *subalternation* between 'No one errs in something' and 'Someone doesn't err in everything'. Caramuel goes on to maintain:

> Etiam sunt disparatae hae *Omnis errat in aliquo*[,] *Aliquis non errat in omni.* Nam esse possunt simul verae, & simul falsae. Verae simul si omnes in aliquo errarent & in aliquo non; falsae simul si nullus erraret (. . .).

'Everyone errs in something', i.e. DQ 3, and 'Someone does not err in everything', i.e. DQ 5, are maintained to be *disparate* in the sense that both propositions can be together true and together false. According to Caramuel they become true in the (realistic) scenario where everybody errs in something but doesn't err in something else: $\forall x \exists y E(x,y) \wedge \forall x \exists y \neg E(x,y)$. Furthermore, in Caramuel's opinion, both become false in a world where nobody ever errs.

Now the first claim certainly is correct because the first conjunct of the scenario guarantees the truth of DQ 3, i.e. $\forall x \exists y E(x,y)$, and the second conjunct entails by subalternation $\exists x \exists y \neg E(x,y)$, i.e. DQ 5. Hence both DQ 3 and DQ5 become true in this case. The second claim, however, is very dubious! The assumption that nobody ever errs just means that DQ 8, i.e. $\forall x \forall y \neg E(x,y)$, is the case. This entails that DQ 3 is false, but it doesn't entail that also DQ 5 would be false. On the contrary, DQ 5 logically follows (by twofold subalternation) from DQ 8![46]

Next Caramuel maintains (quite correctly) that 'Everyone errs in something', i.e. DQ 3, and 'No one errs in something', i.e. DQ 8, are *contrary* to each other. On the one hand, trivially, they can't be together true. On the other hand, they *can* be together false because, in Caramuel's opinion, they *are* together true since at least one man, namely Jesus Christ, never erred in anything (while, clearly, some men sometimes err).

Caramuel concludes his investigations by remarking that 'Someone errs in something', i.e. DQ 4, and 'Someone does not err in something', i.e. DQ 7, are *subcontraries*:

> These two particular propositions can't be together false, but as a matter of fact they are together true; the former because Judas and many other people [occasionally] err; the latter because Christ (and also the Holy Virgin) don't [ever] err.[47]

In the subsequent section 14 Caramuel simply claims that it would be an "easy task to find out how these propositions are related to each other", and he draws the following picture:

[46] Caramuel's tries to support his claim by two rather dubious assumptions: (i) "illud *non* secundae afficeret categorema *Omne* ut tota propositio aequivaleret huic *Aliquis errat in non omni*", and (ii): "qui enim non errare in omnibus dicitur, errare in aliquibus, & in caeteris non errare asseritur."

[47] Cf. *TR*, p. 413, right column: "At vero videntur esse [...] subcontrariae istae duae *Aliquis homo errat in aliquo Aliquis homo non errat in aliquo* [...] istae duae particulares non possunt esse simul falsae, at sunt de facto simul verae, prior, quia Iudas, et alii multi homines errant: posterior, quia Christus, (& etiam Beata Virgo) non errat."


Omnis errat in nullo, | Nullus errat in nullo
Aliquis errat in nullo, | Omnis in aliquo nõ errat | Aliquis non errat in nullo | Nullus in aliquo nõ errat,
Aliquis in aliquo non errat, | Aliquis nõ in aliquo nõ errat.

Figure 6: "Octagon" of opposition (scanned from *TR*, p. 413)

Finally he explains the meaning of these propositions by means of the following equivalences:

Omnis errat in nullo	[$\leftrightarrow$]	Nullus errat in aliquo	[i.e. DQ 8]
Aliquis errat in nullo	[$\leftrightarrow$]	Aliquis non errat in aliquo	[i.e. DQ 7]
Aliquis in aliquo non errat	[$\leftrightarrow$]	Aliquis non errat in aliquo	[i.e. DQ 5]
Nullus errat in nullo	[$\leftrightarrow$]	Omnis errat in aliquo	[i.e. DQ 3]
Aliquis non errat in nullo	[$\leftrightarrow$]	Aliquis errat in aliquo	[i.e. DQ 4]
Nullus in aliquo non errat	[$\leftrightarrow$]	Omnis errat in aliquo	[i.e. DQ 3][48]

The last line, however, contains a mistake. If 'Nullus in aliquo non errat' would be equivalent to 'Omnis errat in aliquo', i.e. to DQ 3, then, according to the fourth line, it would also be equivalent to 'Nullus errat in nullo'. As a matter of fact, however, 'No one in something does not err' is equivalent to 'Everyone errs in everything', i.e. to DQ 1!

Furthermore, Caramuel forgot to explain the meaning of the two propositions 'Omnis in aliquo non errat' and 'Aliquis non in aliquo non errat'. Now in Caramuel's own symbolism explained in section 4.6.2 above, the former proposition has the structure A*IN, i.e. $\forall x \exists y \neg E(x, y)$, and hence it is equivalent to DQ 6. The latter has the structure I*NIN, or I*A, i.e. $\exists x \forall y E(x, y)$, and thus it is equivalent to DQ 2.

Altogether, then, Caramuel's "octagon of opposition" contains all possible propositions with the doubly quantified relation $E(x, y)$ and it becomes comparable to John Buridan's famous octagon discovered almost 300 years earlier.[49] The eight propositions may be analyzed either as forming two "diamonds" or as forming two interlaced squares of opposition.[50] The "diamond" on the left hand side depicts the logical inferences of subalternation between the negative

[48]Cf. *TR*, p. 413, left column.
[49]Cf. Read (2012), pp. 93–110.
[50]This was noted already by Berka & Sousedik (1972), pp. 50–52; and similarly by Dvorak (2008), pp. 645–665.

propositions DQ 8, DQ 7, DQ 6, and DQ 5 (where the implication arrows all point downwards):

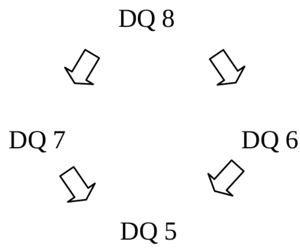

Figure 7:

The "diamond" on the right hand side analogously depicts the logical inferences between the affirmative propositions DQ 3, DQ 4, DQ 1, and DQ 2, where the direction of the implications now, however, points *sidewards* (from right to left):

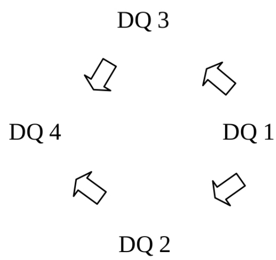

Figure 8:

Moreover, DQ 8, DQ 3, DQ 7, DQ 4 form an ordinary square of opposition:

(A)	DQ 8	DQ 3	(E)
	$\Downarrow$	$\Downarrow$	
(I)	DQ 7	DQ 4	(O)

Figure 9:

Similarly, DQ 6, DQ 1, DQ 5, DQ 2 form another square of opposition:

(A)	DQ 6	DQ 1	(E)
	$\Downarrow$	$\Downarrow$	
(I)	DQ 5	DQ 2	(O).

Figure 10:

Therefore Caramuel's "octagon" may be re-arranged in the following way:

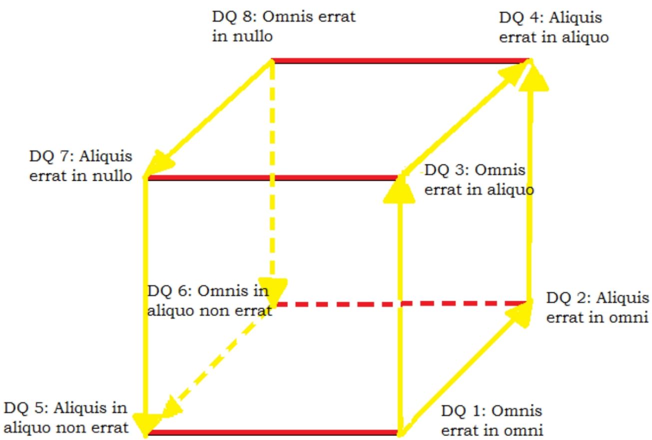

Figure 11: Caramuel's "Cube of opposition"

Light (yellow) arrows symbolize *subalternations*, i.e. logical implications, while dark (red) lines connect propositions which are negations, i.e. *contradictories* of each other. The corners of the top of the cube contain the first square of opposition while the corners of the bottom of the cube contain the second square. The *subcontrary* propositions in these squares are opposed in the direction of the (perspectively) "longer" diagonal while the *contrary* propositions are opposed in the direction of the (perspectively) "shorter" diagonal.[51]

[51]More generally, a relation of *contrariety* obtains between any "negative" proposition p

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(on the left hand side of the cube) and any affirmative proposition (on the right hand side of the cube) provided the latter is logically stronger than $\neg p$. Similarly, contrariety obtains between any affirmative proposition q and any negative proposition which is logically stronger than $\neg q$.

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