

The Genesis of the Paper “Logic without Truth”

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I had the great fortune to work with Carlos Alchourrón for several years, especially while he was associated with the Institute of Juridical Documentation (part of the Italian National Research Council), of which I was Director. In particular, over the period 1984-7 we were able to collaborate under several six-month contracts.

At that time we were very interested in building a legal expert system, but there were important theoretical obstacles in our way, the most important of which is known as Jørgensen’s dilemma.¹ This says that since norms are neither true nor false, we face two alternatives: either (a) logical relationships exist only between statements susceptible of truth or falsehood, in which case no logic of norms is possible, or (b) logical relationships do in fact hold between norms, in which case logic goes beyond pronouncements that can be described as true or false.

One of the major issues that faced us was how to deal with this dilemma. If its first horn is accepted, then it does not seem possible to construct a legal expert system based on logic. Our intuitions lay with the second horn, according to which logic is not confined to statements bearing truth-values, and so could be used in such a project. Of course, the weight of the Aristotelian tradition bears against that option. Aristotle’s very definition of logic was the study of how to obtain further true statements out of given true statements. Could we fight against a tradition more than two millennia old?

We were very much influenced by a famous exchange between Prior and Belnap in the 1960s.² In a paper of only two pages, A.N. Prior had sought to undermine what has come to be called the inferentialist account of logic and thereby defend the essential role of truth-tables in the discipline. He did this by inventing a connective, called *tonk*, which is defined by stating the inference rules that it satisfies but which lacks a truth-table and indeed allows us to deduce an absurdity. In his almost equally brief response, Belnap criticized and, in our view, completely demolished Prior’s argument.

Carlos was in correspondence with Belnap, who suggested that we make use of ideas of Gentzen and Tarski, who defined a basic notion of logical consequence syntactically

¹J. Jørgensen ‘Imperativs and logic’ *Erkenntnis* 1938, 7:288–298.

²A.N. Prior ‘The runabout inference-ticket’ *Analysis* 1960, 21: 38–9, N.D. Belnap ‘Tonk, plonk and plink, *Analysis* 1962, 22: 130–134.

by means of a small system based on axioms and rules. Using the formulation of Gentzen, and writing \vdash for the relation of consequence, the sole axiom and four rules are as follows:

Axiom : $A \vdash A$

Rules :

Weakening: From $A_1, \dots, A_n \vdash C$
infer $A_1, \dots, A_n, B \vdash C$

Permutation: From $A_1, \dots, A_i, A_{i+1}, \dots, A_n \vdash B$
infer $A_1, \dots, A_{i+1}, A_i, \dots, A_n \vdash B$

Contraction: From $A_1, \dots, A_n, A_n \vdash B$
infer $A_1, \dots, A_n \vdash B$

Transitivity: From $A_1, \dots, A_m \vdash B$ and $C_l, \dots, C_n, B \vdash D$
infer $A_1, \dots, A_m, C_l, \dots, C_n \vdash D$.

By using these rules and others added to them, one can define systems of logic that make no reference to the notions of truth and falsehood, indeed do not refer to any semantic interpretation at all. Such independence of logic from semantics provided new possibilities for constructing expert systems.

In this perspective, we sought to build a legal expert system whose propositions take the form of universally quantified conditionals with deontic operators, and an inference motor that consists of inference rules making use of principles of deontic logic that can be applied to a database of legal norms. This strategy was influenced by the pioneering work of Kowalski & Sergot on the law of nationality.

At first we tried to implement our ideas in Prolog, but ran into serious difficulties as a result of the non-classical way in which Prolog treats negation. After some time, we found a group in Milan headed by Stefano Cerri, who was willing to join forces using the programming language Lisp. This collaboration led to a legal reasoning system called SRL.³ Subsequently we were joined by Horacio Arló Costa, a former student of Carlos from the Faculty of Philosophy of the University of Buenos Aires, and constructed a second system called DEO. Both systems were theorem provers capable of dealing with classical negation, and used natural deduction, sequent calculi, and/or tableaux.

In this connection, I would like to recount the following amusing anecdote. When the first theorem prover was ready to go, we arranged a test session one rainy night in the University of Pisa. We posed some queries requiring forward chaining, and some others needing backward chaining, in relation to a common data-base representing the Italian code of family law. For the forward chaining, the machine was asked to generate consequences of the code; for the backward chaining it was asked to find justifications

³C.E. Alchourrón, S.A Cerri, A.A Martino , A. Orsi, D. Santangelo SRL: a legal reasoning system, in A.A. Martino ed., *Expert Systems in Law*, North Holland, 1992.

from the code for specific norms. All went well until Carlos proposed asking the machine to demonstrate a principle of logic without appeal to the database. The information scientists present declared that this was impossible, since computations could only take place on the basis of data. After some rather heated discussion late at night, and just before we had to leave the University, we tried the task and, to the amazement of the pessimists, the machine provided a demonstration of the logical law.

Working with Carlos was quite an affair. We were in Pisa. Every morning I had to travel to Florence to the institute of which I was director. I would get back to Pisa around midday, and take lunch with Carlos – although for him it was breakfast. We would work all afternoon, recording our discussions so as to keep the development and details of our thoughts clear for subsequent reference. Joint work would continue until six or eight in the evening, when people stated going home. But Carlos would continue working, either in the University or walking around Pisa – which was very quiet at night – and never went to bed before three in the morning. A striking feature of his style of thinking is that he would always formulate problems in theoretical terms. A problem that had been raised in the afternoon would be reformulated in a more theoretical manner with a solution often emerging during the night. G.H. von Wright once said that, of all the people he knew, Carlos could plunge himself most deeply into an abstract problem until it became part of his very being.

The paper “Logic without truth”, with Carlos and myself as co-authors, was based on this work in Pisa. The text was composed partly in Pisa, partly in Florence, and partly in Spain where we were invited by Sánchez Mazas. It was published in the journal that Sánchez Mazas edited: *Ratio Juris* 1990, 3:46–67. The whole adventure leading up to the publication was exciting and satisfying, because it simultaneously took us into a highly theoretical and contested field of inquiry, and brought us to construct a theorem-prover that could serve as a legal expert system, with the two tasks constantly interacting.

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