Carlos E. Alchourrón and the Philosophy of Law

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1   Introduction

Carlos Eduardo Alchourrón is one of the most important philosophers of Argentina. His premature death has deprived his country and the whole of philosophical world of an outstanding logician and philosopher.

During many years (from 1955 to 1995) he taught legal philosophy (first as a lecturer, and soon as full professor) at the Law School of Buenos Aires University and after 1983, the year of the restoration of democracy in Argentina with the Presidency of Raul Alfonsin, he became also professor of logic at the Schools of Philosophy and of Exact Sciences. And in all these institutions he had a considerable amount of disciples, who now occupy important positions in different universities in Argentina, in Latin America, Europe and the USA. Alchourrón is well known in many countries, for he participated very actively in numerous meetings and conferences all round the world and was invited professor in the USA, Italy, Spain, Norway, Finland, Sweden and Mexico. He was awarded with several prizes (Guggenheim, Fulbright, Antorchas and Konex), he was a member of Institute International de Philosophie (Paris) and produced several books and numerous papers, published in various international reviews, often written in collaboration with friends and colleagues.

Though his main interest was logic, especially deontic and modal logic in general, he was a great connoisseur of analytic and classical philosophy. His philosophical works have deeply influenced such different, although related, areas as philosophy of law, deontic logic and artificial intelligence. It was his rather unusual competence in all these fields that allowed him to play an outstanding role in international conferences and interdisciplinary meetings. Alchourrón's contributions to all these areas of knowledge are of fundamental importance.

In spite of the fact that logic was his dominant interest, I think that his most important and influential achievements lie in the field of philosophy of law. And as among the participants in this volume, there will be several concerned with logic and artificial intelligence, I think it is convenient to concentrate on Alchourrón's contribution to legal philosophy.

His (temporally) first theoretical interest was music, not so much music as art (though he played flute and sang in a choir), as the theory of music, i.e. the theory of harmony. In spite of the fact that he reluctantly studied, or rather made efforts to study law (for it was the pressure of the family that decided him to choose legal profession) he soon became such a great specialist in harmony that he was appointed professor of harmony at the Conservatorium of Buenos Aires, dependent of the Buenos Aires University. This gave rise to a curious situation: he was at the same time professor of harmony and a (rather bad) student of law.

But his professorship of harmony did not last long: musical theory soon appeared to him as a too narrow field and so he resigned his professorship and turned to logic. A that time modern symbolic logic was almost unknown at the Buenos Aires university, with the sole exception of the Faculty of Exact Sciences where there were several brilliant professors that were excellent logicians, as Julio Rey Pastor (a well known Spanish mathematician, who like many Spanish intellectuals emigrated after Franco's victory in the Spanish civil war), Gregorio Klimovski, specialist in philosophy, logic, physics, and psychology, Mario Bunge, a very well known philosopher of science who later settled in Canada and still works in Montreal, and Rolando García (professor of meteorology), who was the dean of the faculty. Carlos became soon a great specialist in logic, but he continued to work at the Institute of Legal Philosophy. So great was his prestige as philosopher and logician, that as soon as he took his grade in law he was immediately appointed lecturer in legal philosophy. A few years later he became full professor at the Law School.

Alchourrón's rapid academic raise was due not only to his exceptional gifts, but also to professor Ambrosio Gioja, a distinguished scholar who became Director of the Institute for Legal Philosophy and Dean of the Law School in 1955, after the fall of Peron.

Gioja was a prestigious philosopher and an outstanding teacher. He was a follower of Kant and Husserl in general philosophy and of Kelsen in legal philosophy and had collaborated with Kelsen. The meeting of these two so different personalities as Gioja and Alchourrón, a full professor and a recently graduated student, both with important but very different philosophical backgrounds, produced a surprising result: a complete change of the philosophic climate of the Institute. Gioja's original idea was to use phenomenology as the philosophic basis for Kelsen's Pure Theory of Law, whereas Alchourrón pretended to do the same with logic and analytic philosophy. In spite of the difference of age and of academic position, after several years of intense discussions it was Carlos who won the battle: logic occupied a central position and works of Husserl and Kant were replaced by Carnap, Tarski, Wittgenstein and von Wright. In legal philosophy together with Kelsen appeared the works of Alf Ross and H.L.A Hart. This radical change was a great merit not only of Alchourrón, but also of Gioja, who recognized the importance of logic and analytic philosophy.

Among Alchourrón's main contributions to legal philosophy I would mention: 1) Analysis of norms in terms of factual circumstances or cases and deontic qualifications of actions (*solutions*), 2) distinction between generic and individual cases, 3) clarification of such notions as completeness, closure, and consistency, when applied to law, 4) analysis of the notions of a legal system (as a static set of norms) and a legal order (as a dynamic sequence of sets of norms), 5) distinction between norms and norm propositions and analysis of different concepts of permission, 6) analysis of judicial decisions and the problem of relevance and axiological gaps and finally 7) – last but not least – the problem of derogation. A discussion of all these items would largely exceed the space of a paper, so I will concentrate on only some of them.

2   Norms and normative systems

The conception of norms as linguistic expressions in which obligations, prohibitions or permissions of certain actions (or states of affairs resulting from actions) named solutions, are correlated with certain factual circumstances, called cases, is not new, but its analysis especially that of cases in terms of properties, regarded as relevant for the solution of normative problems, is a very important innovation.

A problem is normative if it consists in finding out whether a given action or a set of actions is obligatory, prohibited or permitted. Five universes are used in this analysis: the *Universe of Discourse* (UD), the *Universe of Properties* (UP), the *Universe of Cases* (UC), the *Universe of Actions* (UA) and the *Universe of Solutions* (US).

The Universe of Discourse is the set of all particular situations in which the action(s) in question can be performed. The Universe of Properties is the set of properties that are taken into account for the solution of the problem; these properties can be present or absent in the elements or situations of the UD. Which properties are regarded as relevant is a problem of valuation, but normally this problem is (at least partially) solved by norms enacted by the legislator, or by precedents (former judicial decisions). So we must find out what is prescribed for this problem by legal norms (laws, precedents or customary norms).

The factual range of the problem is constituted by the properties of the UP and by truth-functional compounds of such properties (provided they are not tautological or contradictory); they determine a possible case. If the defining property of a case is a conjunction containing each property of the UP or its negation, but not both, then the case is *elementary* and the set of all elementary cases corresponding to a UP is called *Universe of Cases* (UC). The number of all possible elementary cases corresponding to a UP is easily calculated: it is 2^n, where n is the number of the properties of the UP. So the UC is the set of *all* possible factual situations.

But not all Universes of Cases have their origin in a finite set of properties selected

as relevant. It is rather frequent in legal practice that cases are characterized by numerical values, for instance, in matters of taxes. In such situations a UC may contain an infinite, or at least an unlimited number of cases. This fact makes it convenient to give a general definition of UC: any set of properties which form a division[1], i.e. which are exhaustive of the UD, mutually exclusive and not logically empty is a UC.

As the properties of UP are usually not too numerous (they rarely are more than 4 or 5) the number of elementary cases will also be rather limited in praxis: very rarely more than 16 or 32. This shows that it is relatively easy to determine which all possible factual circumstances (cases) are. Therefore, the worries of many jurists that the legislator cannot foresee all possible future circumstances [2] is absolutely groundless: there is no need for the legislator to foresee anything: his task is not foresight, but decision. He decides (on the basis of evaluations) which properties are relevant for the problem and so classifies all elements of a UD. All other properties are simply irrelevant according to his decision. Of course, it may occur that the valuations of the judge do not coincide with those of the legislator: a property that according to the norms is irrelevant may be deemed relevant by the judge and vice versa: the judge may regard that a property which is relevant for the legislator should not be regarded as such. These kinds of problems give rise to what is called *axiological gaps* that are very different from normative gaps.[3]

In a similar way, we can also calculate which are all possible solutions or possible answers to the problem. As our problem is normative, it concerns the normative status of certain actions. Therefore we must determine the set of actions that can be deontically modalized as obligatory, prohibited or permitted. Some of these actions are basic in the sense that all other actions are truth functional compounds of them. A set of such basic actions is called a *Universe of Actions* (UA). Every action of a UA and every truth functional compound of such actions (provided it is neither tautological nor contradictory) is called *deontic content*. By *solutions* we will understand any deontic content preceded by a deontic operator (obligatory, prohibited, permitted or optional) and its truth functional compounds (which are neither tautological, nor contradictory). If the solution determines all deontic contents of a UA it is called *maximal solution*. The set of all maximal solutions relative to a UA is the *Universe of Maximal Solutions*

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[1]Rudolf Carnap, *Logical Foundations of Probability*, University of Chicago Press, Chicago, 1962: "...a division divides all individuals of the system in question into p kinds or properties which are (a) exhaustive and b) nonoverlapping. Consequently, every one of these individuals must belong to exactly one of the p kinds." p. 108. Cf. C. Alchourrón-E. Bulygin, *Normative Systems*, Springer Verlag, New York- Wien, 1971, 26, henceforth abbreviated as NS.

[2]Cf. G. Demolombe, *Cours de Code Napoleon*, Paris 1880, vol. I, p.136: "En realité, sans doute, les lois ne peuvent pas embrasser toutes les hypothès, si variées, si nombreuses, que chaque jour la pratique soulève." Similar opinion may be found in Savigny (*Vom Beruf unserer Zeit für Gesetzgebung und Rechtswissenschaft*, 1814 and in many other jurisprudential works.

[3]Cf. C. Alchourrón-E. Bulygin, *Normative Systems*, ch. VI, 94–116.

(USmax).

This enables us to discover which of the possible solutions (if any) have been adopted by the normative system we are considering. I will not enter into details of this procedure.

By a *deductive system* we will understand any set of sentences that includes all its logical consequences and if among its consequences there are norms, the system is a *normative system*.

It is important to stress: first, that a normative system may contain not only norms, but also other kinds of sentences, e.g. definitions and conceptual rules in general, as well as political declarations, expressions of purposes, etc., secondly, the sentences constituting the basis of the system may be of different origins, different types and of any number. This makes the notion of a normative system much more flexible, as it is not tied to any legal theory or political ideology.

In the traditional legal theory and in writings of legal philosophers by "legal system" or "legal order" is understood the law as a whole, the totality of all legal norms. But in legal practice, judges, barristers, lawyers, etc. are never interested in the whole legal order. They always deal with some selection (usually rather a small amount) of legal dispositions or norms, which are relevant for the problem they are interested in. In *Normative Systems* by legal system is understood a micro-system, not the whole legal order. I think this is an important innovation. Strangely enough, most critiques of this book with the notorious exception of Riccardo Guastini [4] apparently did not take notice of it.

3   Generic and individual cases

The term "case" is ambiguous, both in ordinary and in legal language. It sometimes is used to refer to certain real events occurring in a definite place and time, like for instance, the murder of Lincoln or of Indira Gandhi. But it can also be used to refer to an abstract entity: a set of properties, as it occurs when we speak of political murder. Political murder is not a real event; it is a mere description applicable to certain events, which may be instantiated in an unlimited number of concrete occurrences. In order to remove this ambiguity we will use the expressions "generic case" and "individual case".

The elements of the Universe of Discourse are individual cases that can be classified by means of certain properties; the result of such classification is a set of generic cases. Generic cases can be instantiated in an unlimited number of occasions, giving rise to individual cases. The classification of individual cases by means of a division gives rise to a Universe of Cases. Such cases are jointly exhaustive of the UD and mutually exclusive. This means that every individual case of the UD necessarily belongs to one and

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[4] R. Guastini, *"Aspetti notevole di Normative Systems"*, *Analisi e Diritto*, 1997, 69–77.

only one generic case of the UC. This property makes the legislation possible. It consists of enactment of general norms which by resolving generic cases provide solutions to all individual cases belonging to them. So the legislative authority can give solutions to an infinite number of individual cases by means of a finite number of general norms.

Judges are in charge of this process: they must resolve individual cases by application of general norms. But in the course of application of general norms to individual cases two difficulties can arise that some authors also call "gaps", though they have little to do with normative gaps. The first problem arises from the lack of information about a relevant fact, the second, from vagueness of descriptive concepts. These problems have been named in Normative Systems "gaps of knowledge" and "gaps of recognition". We use this terminology only because many important jurists use the term "gap" in this connection, but it is important to stress that these gaps are of a very different nature than normative gaps. Normative gaps arise at the conceptual level of generic cases and general norms, gaps of knowledge and gaps of recognition arise at the empirical level of the application of norms to individual cases.

4   Completeness and consistency

An important consequence of the preceding analysis of norms outlined in §2 is the possibility of defining the important notions of completeness and consistency when applied to normative systems. In current legal theory the concept of a system is frequently understood as being necessarily complete (Kelsen, Zittelmann, Cossio) or else necessarily incomplete (Kantorowicz, realist movements). This is a consequence of not taking into account the relational character of the concept of completeness, and of regarding legal orders as wholes, embracing all legal norms, instead of considering each normative system separately.

The concept of completeness is a relation between a normative system (understood as a micro system), a set of possible cases (a UC) and a set of maximal solutions (a USmax): a normative system is complete if each case of the UC is correlated by the normative system to a solution of the USmax. If at least one of the cases is not correlated by the system with a maximal solution, then the system has a normative gap and so is incomplete. This shows that completeness is a contingent property of some systems; there is nothing necessary about it.

The situation is similar regarding consistency: a normative system is inconsistent in relation to a Universe of Cases and a Universe of Solutions if at least one of the cases is correlated to two or more incompatible solutions. Both properties completeness and consistency are contingent.

Some authors distinguish between open and closed systems[5]. A closed system is complete, but its completeness is not relative to any UC, it is absolute. In *Normative Systems* (chapter VI) can be found a rather exhaustive discussion of the different interpretations of this problem and of closure rules. It is sufficient to mention here the main conclusions: (a) the notion of an absolute closure is not applicable to systems containing conditional norms, for it would not preserve consistency, which is one of the adequacy conditions for closure. But legal systems contain always conditional norms, so the concept of absolute closure is not applicable to legal systems. (b) The range of a rule of closure must be restricted: it must be relative to a special universe of cases, the one called *Universe of Relevant Cases* (UCR)[6], that contains all and only relevant properties (relative to a UA and a normative system), as it usually happens in legal contexts with the rule *nullum crimen sine lege.* (3) In spite of this restriction it is important to distinguish between completeness and closedness and between closed and open systems. An open system may be complete in relation to a given UC and a given UA, but incomplete in relation to others. A relatively closed system is necessarily complete in relation to a UCR and to any UA. (4) A well known example of a rule of closure is the rule *nullum crimen sine lege*, as it is frequently used in criminal law: it is important to bear in mind that it is not a norm proposition, but a positive norm permitting all actions not prohibited by penal law and as such it is of course contingent, for it is an expression of a political (liberal) ideology.

5 Norms and norm propositions

Though the distinction between norms, as expressions prescriptive of actions (or states of affairs resulting from actions) and norm propositions descriptive of norms has been mentioned by many philosophers, as for instance, by Bentham [7], Ingmar Hedenius [8], Hans Kelsen [9], Alf Ross [10] and G.H. von Wright [11], it is not easy to characterize it. As an illustration of the difficulty of this problem may be mentioned the fact that such an

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[5]G.H. von Wright, *An Essay in Deontic Logic and the General Theory of Action*, Acta Philosophica Fennica, Fasc. XXI, Helsinki-Amsterdam, 1968, p. 83.

[6]Cf. Normative Systems, 104ff.

[7]J. Bentham (*Of Laws in General*, The Athlone Press, London 1970, edited by H.L.A Hart) distinguished between authoritative and unauthoritative imperatives (152–155).

[8]I. Hedenius, *Om rätt och moral*, Stockholm 1941: genuine and spurious legal sentences.

[9]H. Kelsen, *General Theory of Law and State*, 1946, Reine Rechtslehre 2 ed. Wien, 1962, distinguishes between "Rechtsnorm" and "Rechtssatz" ("legal norm" and "legal sentence", sometimes mistainkingly translated as "legal norm" and "rule of law").

[10]A. Ross, (*On Law and Justice*, Stevens, London 1958, 8–11): distinguishes between propositions of the law and propositions about the law.

[11]G.H. von Wright, *Norm and Action*, Routledge and Kegan Paul, London, 1963, VIII, 110.

acute thinker as von Wright made several attempts to tackle it, without getting a fully satisfactory solution. Several years later (1998) he wrote referring to this distinction:"It has taken me nearly thirty years to see its full significance. And not all logicians and philosophers have seen it even now." [12]

The main difficulty lies in the fact that traditionally (i.e. since Aristotle) such concepts as logical consequence and consistency, as well as logical connectives, like negation, or conjunction are defined in terms of truth. So it seems that only expressions capable of being true or false can enter into logical relations and so – if one thinks that norms lack truth values – logic of norms seems to be impossible. On the other hand, in ordinary language appear connectives like 'and', 'or' 'no', etc., apparently with the same meaning as in descriptive discourse and there are inferences with normative premises or conclusions and such inferences seem to be logically valid. So there must be a logic of norms. This problem is known in the literature as Jørgensen's dilemma and since its formulation in 1938 by the Danish logician Jørg Jørgensen has been intensely discussed.

In von Wright's first paper on deontic logic[13] the deontic formulas were treated as true or false propositions. Some years later in the Preface to his *Logical Studies*[14] he recognized that this approach was"philosophically unsatisfactory" for the simple reason that norms lack truth values. The conclusion he drew was that"logic has a wider reach than truth", but he did not develop this idea. In *Norm and Action* (1963) he draws a clear distinction between norms and norm propositions. Though in ordinary language norms and norm propositions can be expressed by similar or even by the same words, von Wright recognizes that they are quite different in nature. But he uses for both entities the same symbolism with two interpretations[15]: a prescriptive and a descriptive one, which for various reasons was also not a satisfactory solution of the problem.

Since his first visit to Argentina in 1968 there was a continuous exchange of ideas between him and Alchourrón on many topics, but especially on normative logic. Though they never published a joint work there was a permanent dialogue between them. In 1969 Alchourrón published an important paper[16], in which he develops a separate logic for norm propositions, using different symbolism.

The main formal differences between the logic of norms (henceforth LN) and the logic of norm propositions (LNP) are: 1) norm propositions are relative to sets of norms (normative systems), hence the formulas of LNP are stating that a certain norm belongs to a given normative system: either as '$Op \in S$', or by means of subscripts ($O_S p$,

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[12]G.H. von Wright, "Value, Norm, and Action in My Philosophical Writings", in Georg Meggle (ed.), *Actions, Norms, Values. Discussions with Georg Henrik von Wright*, Walter de Gruyter, Berlin - New York, 1999, 20.

[13]G.H. von Wright, "Deontic Logic", *Mind* 1, 1951.

[14]G.H. von Wright, *Logical Studies*, Routledge and Kegan Paul, London,1957, VII.

[15]*Norm and Action*, 132.

[16]Carlos E. Alchourrón, "Logic of norms and Logic of normative propositions", *Logique and Analyse* 47 (1969), 242–268.

meaning: the action p is obligatory according to the system S), whereas norms like 'Op' are not relative, but absolute concepts. 2) Negation plays a very different role in LN ad LNP. Negation in LN is similar to the negation in propositional logic: a) in the same way as the negation of a proposition is a proposition, the negation of a norm is a norm; b) there is only one negation of a given proposition (norm); c) a proposition (norm) and its negation are reciprocal and (d) propositions (norms) and their negations are mutually exclusive and jointly exhaustive. But in LNP there are two kinds of negation, an *external negation* that operates over the membership of the norm in the normative system in question and an *internal negation* operating over the norm. Therefore the expression "p is not prohibited in S" becomes ambiguous: it can mean "the norm prohibiting p does not belong to S" (external negation), or, "the negation norm of "p is prohibited" belongs to S". But the negation norm of "p is prohibited" is the norm permitting p. In symbols: the internal negation of '$O{\sim}p \in S$' is '${\sim}O{\sim}p \in S$', where '${\sim}O{\sim}p$' is equivalent to 'Pp'. Only in the limiting case when the normative system in question is complete and consistent, the difference between internal and external negation vanishes and the LNP becomes isomorphic with LN.

In Alchourrón's 1969 paper von Wright's first system of deontic logic (1951) is regarded as a substantially correct reconstruction of a logic of norms, which means that there are logical relations between norms, even if norms lack truth values. But there is no explicit justification of this idea, and so the problem posed by Jørgensen's dilemma persists.

In a paper published in 1983[17] von Wright made a new attempt to solve this problem by constructing a LN without appealing to truth. This logic is based on the incompatibility of certain acts of norm-giving, as the acts of commanding p – e.g. to open a window – and the act of commanding ${\sim}p$, i.e. to leave it closed. Such acts are incompatible because the norms issued by them cannot be obeyed. Even if it is possible that a norm authority issues such commands one would say that it behaves irrationally, for the person that issues a command normally wants that the commanded action be performed, which is logically impossible in such cases. So this logic of norms is built on the idea of rationality of the norm giving activity, i.e. the activity of enacting norms. Such relations between certain acts are not strictly logical in the traditional sense: for there are no logical relations between acts. Only in an extended sense can such relations between acts be regarded as logical. [18]

In a paper published in 1984[19] we (i.e. Carlos and me) explicitly accepted von

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[17]G.H. von Wright, "*Norms*, Truth, and Logic" in *Practical Reason*, Basil Blackwell, Oxford 1983, 130–209.

[18]Jorge L. Rodríguez has suggested in a private communication that though there are no logical relations between acts, there can be logical relations between the results of certain acts. In which case norms would not be prescriptions, but results of acts of prescribing.

[19]C.E. Alchourrón and E. Bulygin, "Pragmatic Foundations for a Logic of Norms", *Rechtstheorie* 15 (1984), 453–464.

Wright's ideas about rationality of norm giving authority as a basis for a logic of norms. So the logic of norms (LN) becomes a reconstruction of the rationality of the legislative activity, whereas the logic of norm propositions (LNP) is concerned with the reconstruction of the logical consequences of a given set of norms (a normative system).

The next step in this philosophic dialogue was Alchourrón's idea to elaborate in details von Wright's suggestion that "logic has a wider reach than truth", by resorting to a general (abstract) notion of logical consequence, which is neither syntactic nor semantic. Using some ideas of Gentzen and Belnap, Alchourrón develops a calculus for LN without recurring to truth. This would yield a more solid foundation for a logic of norms. [20]

6   The importance of the distinction between norms and norm propositions.

The main importance of the logic of norm propositions is that it allows formulating clearly several notions obscurely dealt with by many legal philosophers, like the concepts of permission and the famous problem of legal gaps.

Many legal philosophers and deontic logicians maintain the view that permission is mere absence of a prohibition: this view leads to the conclusion that all normative systems are complete in the sense that they necessarily qualify every possible action as prohibited or permitted. The argument is very simple: every action is either prohibited or not prohibited and if 'permitted' means 'not prohibited', then every action is either prohibited or permitted. Therefore every action is deontically qualified (as prohibited or permitted) by any set of norms or normative system, independently of its contents. Hence the conclusion that all legal systems are necessarily complete in the sense that they regulate all possible actions, i.e. that there are no gaps in the law. This idea has been sometimes maintained even by von Wright[21] and it can be found in such important and otherwise quite different legal philosophers as Hans Kelsen, Joseph Raz and Ronald Dworkin. [22]

It is relatively easy to show that this idea is wrong. We have already seen that the

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[20]Pablo Navarro and Jorge Rodríguez accept Alchourrón's solution of this intricate problem. Cf. their excellent book *Deontic Logic and Legal Systems*, Cambridge University Press, New York, 2014.

[21]G.H. von Wright, *A Essay in Deontic Logic and the General Theory of Action*, Acta Philosophica Fennica XXI (1968), North Holland: "A normative system is closed when every action is deontically determined in this system" (83) and "Any system of norms is therefore closed, of which it holds true that, for all values of variables, either a norm to the effect that $P(p/q)$ or the norm to the effect that $\sim O(p/q)$ belongs to this system." (84).

[22]E. Bulygin, "Kelsen on the Completeness and Consistency of Law" in Luis Duarte d'Almeida, John Gardner and Leslie Green (eds.), *Kelsen Revisited. New Essays on the Pure Theory of Law*, Hart Publishing, Oxford, 2013, 225–243.

expression "not prohibited" is ambiguous. If the negation is external, then "not prohibited" means "the prohibition of p does not belong to S" ($O\sim p \notin S$) and the dictum every action is either prohibited or permitted is a mere tautology, for it only means that what is not prohibited is not prohibited. Permission as absence of prohibition is usually called *negative permission*. It certainly cannot be used to prove that the law is necessarily complete and that legal gaps are impossible. If the negation is internal, then "p is not prohibited in S" means "p is permitted in S", i.e. the norm permitting p belongs to S ($\sim O\sim p \in S$). This is another meaning of "permitted": i.e. *positive permission*. Once we distinguish between negative and positive permission it becomes clear that neither version can support the idea that all normative systems are necessarily (for conceptual reasons) complete and there are no gaps in the law. Negative permission is mere absence of a norm, but positive permission requires a (permissive) norm. Such a norm cannot be inferred from the mere fact that there is no norm prohibiting p. As it happens with all (positive, i.e man made) norms its existence requires a normative fact (legislative decision, custom or precedent). And in the same way as there are no prohibitions without a prohibitive norm, there cannot be a positive permission without a permissive norm. And its existence is contingent: for there are no necessary man made norms.

But are there permissive norm? This question has been answered negatively by many prominent philosophers. The main argument against permissive norm is that only imperative norms, i.e. norms establishing obligations and prohibitions are capable of guiding human behavior; permission is compatible with any conduct of the addressee of the norm and so it does not guide any action. [23]

It is true that a permissive norm cannot be violated and so is compatible with any conduct of its addressee. But this does not mean that permissive norms are altogether irrelevant. They play an important role in the context of a normative system. In the first place, they are used for eliminating or suppressing (by derogation) an imperative norm. Secondly, they can limit the competence of a hierarchically inferior authority: if an action has been permitted by a higher authority, it cannot be prohibited by a lower authority. So it is by no means always senseless to permit a hitherto not prohibited action. Here it is important to distinguish between merely not prohibited actions (which belong to the sphere of original permissibility) and those actions that are permitted by an act of authority (supervening or positive permissibility).[24] Therefore permissive norms fulfill important tasks in the context of a normative system: they are by no means superfluous or irrelevant and cannot be reduced to imperative norms. The

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[23]D.T. Echave - M.E. Urquijo - R.A. Guibourg, *Lógica, proposición y norma*, Astrea, Buenos Aires 1980, M. Atienza - J. Ruiz Manero, Las piezas del derecho, Ariel, Barcelona 1986.

[24]E. Bulygin, "Permissive Norms and Normative Systems", in A.A. Martino - F. Socci Natale (eds.) *Automated Analysis of Legal Texts*, North Holland, 1986, 211–218 and Eugenio Bulygin, *Essays in Legal Philosophy* (1984/86/2012. Written with Carlos E. Alchourrón, Oxford University Press, Oxford 2015, 324–336.

fact that they are not necessarily expressed in terms of permissions, but can also be expressed in terms of obligation, prohibition, rejection or derogation does not change their permissive character. [25]

7   Analogies with valuations and preferences

A second point that illustrates the importance of the distinction between norms and norm propositions are the striking analogies in other fields, specifically in valuations and preferences.

(a) It is important to observe that value terms, like "good", "right", "reasonable" etc. play different roles in different contexts: they can be used to express valuations or to state that in a given case the conditions required in order that something be regarded as valuable (good or right) are satisfied. In this second case the sentence 'x is good' may not express any approval: in such case it is not a valuation, but a mere factual description. This allows distinguishing between *value judgments*, which are expressive of valuations, and what can conveniently be named *axiological propositions* – which in spite of the occurrence of value-terms – do not express valuations, but are purely descriptive and hence true or false. The situation is analogous to norms and norm propositions. In both cases, we have expressive sentences (expressing either norms or valuations), which lack truth values, and on the other hand, true or false descriptive propositions (normative or axiological). The logical behavior of norm propositions and of axiological propositions is similar. On close examination it appears that in legal contexts there are much less valuations as is usually believed by judges or practical lawyers. This distinction between value judgments and axiological propositions has been analyzed in a joint paper by Alchourrón and Bulygin, [26] where we gave several examples of problems frequently regarded as valuations, and which are factual (empirical), in spite of the use of typical value terms like good or right.

(b) Another interesting analogy can be traced between sentences expressing preferences and propositions about such preferences (*preferential propositions*). This analogy has been thoroughly discussed in a very important book by Jorge L. Rodriguez. [27] Rodríguez has convincingly shown that von Wright's *The Logic of Preference* [28] deals with preferences and not with preferential propositions. So he develops an interesting

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[25]E. Bulygin, "Sobre la equivalencia pragmática entre permiso y no prohibición" *Doxa* 33 (2010), 283–296.

[26]C. Alchourrón- E. Bulygin, "Limits of Logic and Legal Reasoning" in A.A. Martino (ed.) *Preproceedings of the III International Conference on Logica Informatica*, Diritto, vol.II, Firenze, 1989.

[27]Jorge L.Rodríguez, *Lógica de los sistemas jurídicos*, Centro de Estudios Políticos y Constitucionales, Madrid 2002, 211–247.

[28]G.H. von Wright, *The Logic of Preference*, Edinburgh University Press, Edinburgh, 1963.

complementary analysis of preferential propositions, showing that there is a very strong analogy between preferential propositions and norm propositions.

In all three areas we find the same phenomenon: there are sentences expressive of certain pragmatic attitudes (norms, value judgments and preferences), which lack truth values, and there are true or false propositions about the outcome of such sentences, i.e. the logical consequences of sets of norms, of value judgments or of preferences. These consequences are described by norm propositions, axiological propositions and preferential propositions. And so we have (or rather should have) three pairs of logics: 1) logic of norms and logic of norm propositions, 2) logic of value judgments and logic of axiological propositions, and 3) logic of preferences and logic of preferential propositions.

The problems so far discussed in this paper have been mainly analyzed in *Normative Systems*, a book that in spite of its considerable age (the English version was published in 1971) is still read and discussed in many countries. [29]

But there are many important problems that have been analyzed in subsequent years, of which only two will be shortly commented here.

8   Legal System and Legal Order

The idea of a legal system referred to a given temporal moment, called *momentary legal system*, was introduced by Joseph Raz[30], who defines it as the set of all laws (i.e. legal norms) valid at a certain temporal moment, as opposed to *legal system*, which contains all the laws of a country; a momentary legal system is a static notion: it cannot change, for any change of its contents leads to another momentary system. The laws of a country change with time; it is a dynamic notion.

We have tackled this problem in *Normative Systems*, introducing the terminological distinction between *legal system*, corresponding to Razian momentary system, and *legal order* presumably corresponding to his legal system. I say "presumably", because it is not clear whether what Raz calls legal system is the set of momentary systems or

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[29]The Spanish (original) version was published in 1975 under the title *Introducción a la Metodología de las Ciencias Jurídicas y Sociales* by Astrea, Buenos Aires, the German translation appeared in 1994 (*Normative Systeme*, Verlag Karl Alber, Freiburg/München), and then was followed by the Italian translation (*Sistemi normativi. Introduzione alla metodologia della scienza giuridica*, G. Giappichelli editor, Torino 2005), and the Russian translation (first published in *Russian Annuary for Legal Theory* No.3. 2010, 309–472, and later together with other works of the authors and other collaborators by the State University of St. Petersburg, Saint Petersburg, 2013. A new Spanish edition (after 5 reimpressions) was published by Astrea in 2012 with the title of *SISTEMAS NORMATIVOS. Introducción a la metodología de las ciencias jurídicas*.

[30]J. Raz, The Concept of a Legal System. *An Introduction to the Theory of Legal Systems*, Clarendon Press, Oxford 1970, 34 ff.

the set of norms that belong to all momentary systems. In any case, by legal order we understand the set of legal systems, i.e. a family (a temporal sequence) of systems. So legal system is a static notion, whereas the concept of a legal order is dynamic. This topic was developed in several papers published after *Normative Systems*[31].

Two main problems arise in this connection: the structure and the identity criterion. The first problem consists of determining the contents of a legal system, corresponding to a definite temporal moment; the second refers to the identity criteria of a legal order. Both problems concern the criteria of membership: membership of norms to a system and membership of systems to an order. In the legal domain, two criteria are generally used in this connection: deducibility and legality. According to the criterion of deducibility a norm belongs to a system if it is a logical consequence of (deducible from) the norms of this system. According to the criterion of legality a norm belongs to a system if it has been issued by a competent authority of the system. An authority x is competent to issue a norm N if there is a norm belonging to the system that authorizes (or empowers) x to issue N. Therefore there are two types of relation between norms: logical implication and legality. These relations determine the structure of the system.[32]

I prefer to formulate the criteria of membership of norms and of systems in terms of validity. The term "validity" is notoriously ambiguous in legal parlance: it can mean existence, binding force or membership. I will use it in this last sense.

The criteria of deducibility and legality cannot give account of the validity or membership of all norms to a system, because they already presuppose the existence of norms whose validity does not depend of these two criteria. Such norms are called *independent* (Caracciolo) or *sovereign* (von Wright, 1963, 199).

Every legal order must be originated by a system of independent or sovereign norms which may be named the *original system*. Its norms belong to all subsequent systems; all systems belonging to this order (except the first, i.e. the original system) originate in the temporally preceding system by means of addition (as a consequence of acts of enacting new norms) or subtraction (as a consequence of acts of derogation).

We can now formulate a criterion of identification of a legal order O in terms of validity (membership) in a simplified version, by means of four rules:

(1) The set of norms $\{N_1, N_2, \ldots, N_n\}$ is the original system of O.

(2) All norms enacted at time t by a competent authority of O are valid norms of all systems (which are members of O) after t, unless they are derogated later by a competent authority.

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[31]C.E. Alchourrón - E.Bulygin, "Sobre el concepto de orden jurídico", Crítica. *Revista Hispanoamericana de Filosofía*, vol.6 No.23, 3–23, E. Bulygin, "Algunas consideraciones sobre los sistemas jurídicos", *Doxa* 9, 257–279.

[32]R.A. Caracciolo, Sistema jurídico. *Problemas actuales*, Centro de Estudios Constitucionales, Madrid, 1988.

(3) If a norm is derogated by a competent authority at time t, it ceases to be valid, i.e. it does not belong to subsequent systems, unless it is promulgated again by a competent authority.

(4) All norms that are logical consequences of (deducible from) norms that belong to a system are valid norms of that system.

The rule (1) identifies the order O: other legal orders differ from O not by their structure (which is the same), but by its original constitution. This constitution may contain any kind of norms; the only requirement is that it must contain at least one norm of competence, empowering an authority (a person or a body) to enact (and eventually to derogate) valid norms. The norms mentioned in rule (1) belong to the order by definition: (1) is a mere enumeration of independent norms.

The rules (2) and (3) are responsible for the dynamic nature of the order O, making possible its change. The rule (4) determines that all systems belonging to O are deductive systems. The rules (2) and (3) define the membership of the systems to the order and determine the contents of each system. The rules (1) - (4) form a recursive definition of "a norm valid in a system belonging to O", for by means of successive application of these rules it is possible (in a finite number of steps) to determine whether a given norm is a member of (is valid in) a given system of a given legal order.

Though the contents of a given system depend on normative acts (acts of promulgation and derogation) performed by competent authorities, which is an empirical question, to find out which norms are added to the system or eliminated from it by such acts raises complicated logical problems that will not be discussed here. [33]

9   Derogation

The derogation (suppression or withdrawing) of legal norms is a well known phenomenon in legal sphere, but its logical implications have not been studied by jurists and legal philosophers. Carlos and me began to investigate it in early seventies and in 1976 we presented a paper in a conference in Bielefeld (published in German and in Italian[34]). A few years later Alchourrón and David Makinson published an important

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[33] Alchourrón-Bulygin, "Sobre el concepto de orden jurídico", cit. in FN 29 and *Sobre la existencia de las Normas Jurídicas*, Valencia, Venezuela (1979), reproduced by Fontamara, México, 1997. *Normas Jurídicas*, Valencia (Venezuela), 1971.

[34] "Unvollständigkeit, Widerspüchlichkeit und Unbestimmtheit der Normenordnungen", A.G. Conte-R. Hilpinen-G.H. von Wright (eds.), *Deontische Logik und Semantik*, Athenaion, Wiesbaden 1977; "Incompletezza, contraddittorietà e indeterminatezza degli ordinamenti normativi" in G. di Bernardo (ed.), *Logica deontica e semantica*, il Mulino, Bologna 1977.

paper on"Hierarchies of Regulations" [35] in which they investigated how to overcome the indeterminacy resulting from derogation by means of imposing a partial ordering on the set of norms (regulations) and they showed how this method can be used to confer uniqueness upon indefinite derogations.

In the course of their investigations they discovered that derogation of legal norms is a particular case of a much more general problem. In this way the particular problem of derogation of norms gave rise to a general theory of elimination of certain not desirable components or consequences of a given theory. This discovery led to an intense collaboration not only between Alchourrón and Makinson, but also with Peter Gärdenfors who came to this problem independently. The result of the collaboration of this triad was the famous paper that appeared in 1985[36] and is widely known as paradigm AGM. It is a classic, not only in deontic logic and artificial intelligence, but also in legal philosophy.

XXX

Carlos Alchourrón has been during many years my close friend and we have worked together for almost forty years and have written several books and a lot of papers. I would like to stress that all the ideas, including those that can be found not only in our joined publications, but also in those written by me alone, even after Carlos' death, have been deeply influenced by his thoughts.

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[35]C. Alchourrón/D. Makinson, "Hierachies of Regulations and Their Logic", in R. Hilpinen (ed.), *New Studies in Deontic Logic*, Reidel, Dordrecht 1981, 125–148.

[36]C.E. Alchourrón-P. Gärdenfors-D. Makinson, "On the Logic of Theory Change: Partial Meet Contraction and Revision Functions", *The Journal of Symbolic Logic*, vol. 50, Number 2, June 1985, 510–530.