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Epistemic-Temporal Logic and Sortal Predicates

Max A. Freund

Abstract

A formal semantics for epistemic-tense formal languages for sortal predicates is formulated. Conceptualism is the philosophical background of this semantic system. Completeness and soundness theorems are proved for a restriction of the semantics. The restriction is motivated by a nativist view of concept-formation.

Keywords: epistemic logic, tense logic, logic of sortal predicates, relative identity, relative quantication, conceptualism, nativism, second-order logic.

1 Introduction

The syntax of classical first-order logic does not distinguish monadic predicates that provides criteria of counting, identity and classification from those that do not. Examples of the former kind of predicates are most of our common nouns (such as "spider" and "horse")[1]. Adjectives and mass terms, on the other hand, are clear cases of predicates not supplying all of the three sort of criteria at the same time. Although mass predicates, like "water" for example, gives criteria for identity and classification, they do not do it for counting. Adjectives provides criteria for classification but not for identity and counting. Predicates providing all of the three criteria in question are known as "sortal predicates".

Sortal predicates have important logico-linguistic properties that taken together would differentiate them from other kinds of predicates. These properties are byproducts of the criteria provided by sortal predicates. Such properties include the possibility of forming relative identities, relative quantifications and predications. In other words, with sortal predicates we can form relative (sortal) identities such as "a is the same spider as b" and "a is the same horse as b", and relative (sortal) quantifications, such as "every spider" and "some horse". In addition to these features, sortal predicates can occur in predications. For example, regarding the sortal predicate "person", one can assert that *John is a person*. In general, for any sortal predicate S, it is meaningful to claim that *a is an* S. We should note that by means of mass terms relative identities can be formed, but not relative quantifications and predications. With adjectives, we can construct predications, but not relative identities or relative quantifications.

Now, when a formal language allows for the formal representation of sortal quantification and identity, and it is such that its logical syntax assumes such representations as undefinable, we shall speak here of *a (formal) language for sortal predicates* [2]. In previous papers, we have studied extensional and intensional languages for sortal terms (see Freund (2007),(2004), (2002) and (2001)) and constructed semantic systems for such languages. These papers have assumed a modern form of conceptualism (as a theory of universals) as part of the philosophical background theory for those systems. With the exception of sentences, such a philosophical theory presupposes, in general, that meaningful linguistic expressions stand for concepts and constitute their semantic basis. Thus, in particular, for conceptualism, sortal predicates have to be assumed to represent concepts. We shall refer to this sort of concepts as sortal concepts.

Another main feature of the above modern version of conceptualism is its view of concepts as cognitive capacities or structures based on such capacities. As such, the set of concepts might vary from one time-point to another. Also, if possible worlds are taken into consideration, the set in question can also vary through possible worlds. Now, since construction of knowledge depends on the set of concepts we might possess, a clear link between the development of knowledge and the development of concept-formation can be presumed. In other words, a connection between the dynamics of knowledge and the dynamics of concept formation can be clearly supposed.

Now, the above link between knowledge and concept-formation suggests different epistemic notions. We are here interested in one of these possible notions, namely: the notion of knowability relative to development of sortal concepts. Our goal, in this paper, is to inquiry into the logic of such a notion, within the context of a formal sortal language. Since the notion of development is being involved in the notion itself, we shall here take into account the dimension of temporality. Thus, the language for sortals characterized in this paper will also include temporal propositional conectives. In other words, the formal language for sortals we shall here consider will be two dimensional: epistemic and temporal. The language will also be second-order regarding sortal term variables, that is, the logical syntax of the language will allow for the concatenation of universal quantifiers with sortal term variables. This syntactic feature together with its semantic interpretation will be an important part of the representation of the dynamic factor in conceptual development.

Now, regarding the development of concepts, two sort of theories will be in perspective in the present paper, namely: radical nativist, on the one hand, and partial and non-nativist theories of concept-formation, on the other hand. A radical nativist theory assumes that all of our concepts are innate or formed from innate concepts by logical and other mental operations. The role of the environment is just instrumental in the activation of those cognitive capacities. But no other role is assigned to the environment in the content of the concepts themselves. Partial nativist and non-nativist theories have a different conception. For partial nativism, there clearly are innate concepts, but also there are concepts that are not. These latter ones get formed from the interaction of the individuals with their environment. According to the non-nativist theories, all of the concepts are not innate and are a byproduct of the experience of the individuals with the environment. The environment plays an essential role in the content of the concepts.

The semantics we shall here formulate might be viewed as formally representing different aspects of the non-nativist theories regarding the epistemic notion of *knowability relative to the development of sortal concepts*. However, there is a subclass of the models of the semantics fulfilling certain conditions, which might be justified by appealing to a radical nativist-view of concept-formation. We shall prove soundness and completeness with respect to the notion of logical validity provided by that subclass. Soundness and completeness proofs for the semantics in general is left as an open problem.

As the readers will notice, the semantics provides a variable-domain interpretation of the second-order quantifiers over sortals, that is, the range of such quantifiers at a possible world and time point will be the set of concepts that have been formed at that world and time point only. This is in consonance with one of the aspects in the philosophical motivation of the paper, namely: the variation of the set of concepts through time and possible worlds as a factor in the determination of knowledge. A variable-domain interpretation of second-order quantification is the interpretation more in accord with such a motivation.

2 Philosophical preliminaries

As indicated, we shall assume a modern version of conceptualism as philosophical background. In the present section, we shall briefly present some features of this philosophical theory. These features are relevant for the philosophical justification of the semantics and syntax of the epistemic logical system for sortals here characterized [3].

In general, in contrast with nominalism and realism, conceptualism as a theory of universals postulates concepts as the semantic grounds for general terms. In the modern version of conceptualism assumed in this paper, this view is extended to all meaningful linguistic expressions other than sentences. Also, in this contemporary variant of conceptualism, there is an interpretation of concepts as cognitive capacities or structures based on such capacities. Depending on the nature of such capacities or structures different sorts of concepts may be distinguished.

We begin by differentiating between *sortal* and *predicable concepts*, the former being intersubjectively realizable cognitive capacities whose uses in thought and communication are associated with certain criteria by which we are able to distinguish, count, identify and classify objects. Clearly, sortal predicates will represent sortal concepts. So, for example, the predicate “house” will stand for a cognitive capacity that allow us to count, identify and classify houses.

Predicable concepts, on the other hand, are intersubjectively realizable cognitive capacities enabling us to classify and relate objects. Non-sortal monadic predicates terms and n-place predicate expressions, for each $n > 1$ (also known as relational predicates), will stand for *predicable concepts*. Thus, terms like “red”, “run” and “smaller than” will represent predicable concepts.

Apart from sortal and predicable concepts, there is another sort of concepts relevant for the topic of the present paper. We shall refer to them as *referential concepts*. Their exercise allow us to refer. But here again we can make distinctions.

There are referential concepts based solely on sortal concepts. Cases of this kind of concepts constitute concepts represented by expressions of the form “every S” and “some S” (where S is a sortal predicate expression), such as “every horse” and “some houses”. In general, referential concepts based on sortal concepts are intersubjectively realizable cognitive structures that enable us to refer to objects distinguished and classified by sortal concepts.

Second-order referential constitute another kind of referential concepts, in particular, those whose exercise allows us to refer to the sortal concepts themselves. Linguistic expressions such as “every sortal concept” and “some sortal concept” stand for concepts of this kind. Now, two different interpretations can be provided to these sort of expressions, namely: a counterfactual and a variable-domain interpretation. That is, second-order reference to sortal concepts can be understood as reference to all possible concepts or only to the concepts constructed at the time-interval and possible world at which the referential concept is being exercised.

We shall here adopt the variable-domain interpretation. This is because our goal is to construct an epistemic logic that will take into account the determination of knowledge through time and/or possible worlds by the formation of sortal concepts. A variable-domain interpretation will formally represent much better the dynamics of such a concept-construction. The interpretation in question will be an aid in the formal representation of the link between the dynamics of knowledge and the dynamics of concept formation.

We now wish to consider the notion that constitutes the focus of our paper. This is the notion of what in principle can be known relative to the sets of constructed sortal concepts or, in other words, knowability relative to the development of sortal concepts. Formation of these concepts at a time interval impiges on the content of knowledge we could have access to at that interval. This is due to the fact that propositions, as they are viewed in the present philosophical theory, are formed out of concepts and, in particular, sortal concepts. Thus, for example, at any time-point at which the sortal concept of being a house is being formed, certain propositions involving such a concept can be possibly constructed. For instance, propositions involving relative sortal quantification over houses will contain the concept of being a house, such as the proposition that some houses are white. This proposition is composed out of a referential concept (the concept represented by "some house") and the predicable concept of being white. Construction of that proposition and many others presuppose formation of the sortal concept of being a house.

In accordance with the above, the continuing transformation of the set of sortal concepts at different time-intervals might be a factor determining the propositions that in principle can be constructed at different temporal points. But this, at the same time, might imply a transformation of what can be known in principle at a period of time: for an agent to know in principle that p, s/he must in principle believe that p. But for the agent to believe in principle that p, an awareness of p must be possible and for this to occur within the present conceptualist framework, a construction of the proposition p by the agent is a necessary condition. Thus, the set of propositions that in principle can be validated as knowledge at a time-point will be a subset of the propositions that in principle can be constructed out of the concepts formed at the time-point in question. Consequently, transformation of the set of sortal concepts might imply transformation of what can be known in principle.

Given the above, from now on we shall refer to a subclass of propositions as *an epistemic matrix of a time-point* when it fulfills the following: (1) they can in principle be constructed out of the sortal and other kind of concepts formed at that point and (2) they can in principle be validated as knowledge. It is a matrix because such a subclass is the source from which knowledge at that time-point could arise. Thus, any proposition constructed by a agent at a time-interval on the basis of the sortal concepts formed and validated by the agent as knowledge, at that interval, will belong to the epistemic matrix at that period of time. In this idea of validation, we are not assuming a particular philosophical conception of epistemic validation.

Clearly, the continuing change of the set of sortal concepts might imply a constant change of the epistemic matrix. That is, at different time points we might get different epistemic matrices. Correlative to the formation of sortal concepts at a time-point, we get a group of possible propositions that in principle can be formed at that time-point. Within this group of propositions, we get a class of possible propositions that could in principle be validated as knowledge.

We should point out that, in the present paper, we shall not discuss or logically explore the notion of a possible construction of a proposition as presupposed in the above explanations. The same applies to the notions of awareness and belief of a proposition. We shall here assume these different notions as basic or primitive. We shall also note that we are not assuming that there is an *epistemic matrix* for every time-interval. Since the knowing agent is one of the essential factor in the formation of concepts and propositions, unexistence of the agent at a time-point implies unexistence of an epistemic matrix at that point.

Now, the notion of an epistemic matrix for the actual world at a time-point suggests the idea of worlds alternative to that world sharing the same epistemic matrix at the same point. That is, we can envisage the class of worlds different from ours (in many different ways) but having in common at a given time-point the same epistemic matrix as that of the actual world at the same time-point. We can generalize this idea by considering possible worlds in general and their associated temporal matrices. Knowability (or knowledge in principle) relative to sortal concepts constructed at a given time-point, with respect to a possible world (actual or otherwise), will amount to truth in all alternative possible worlds that share the same epistemic matrix at the same time-point. For this interpretation of relative knowability, it is clear then that epistemic versions of the S5 modal system is the adequate system of epistemic logical principles.

We want now to consider another important philosophical point. This is question of the role of the environment in the development of concepts. We shall focus here on two kinds of answers to this question. One of this two answers upholds the idea that all of our concepts are innate or derived from innate concepts by mental operations (including logical operations among them). The role of the environment is only instrumental in triggering those concepts. This is called radical nativism. A contemporary version of this philosophical approach can be found, for example, in Fodor (1975) and (1981) (see Fodor (2008) for an assessment of his earlier views on nativism). Fodor's theory would seem to imply that sortal concepts are innate. He explicitly states, for example, that the concept of being a doorknob is innate.

The other kind of answers are constituted by either partial nativist or non-nativist theories of concept-formation. The former sort of theories allows for some innate concepts, but also for other concepts in which the environment does not reduce to the instrumental role of awaking the concepts. Such concepts do not exist previously to the interaction with the environment. Kantian oriented epistemologies are usually of this sort of nativism. Non-nativist theories clearly assumes that all concepts are derived from experience. Empiricist philosophies are generally committed to this sort of approach.

In the present paper, we shall not argue in favor or against any of the above theories, nativist or otherwise. We shall try to represent formally some elements of the non-nativist and the radical nativist theories as far as the epistemic operator of this paper is concerned. The semantics we shall formulate in the next section can be justified on the basis of a non-nativist conception. The set of sortal concepts and the knowledge in principle as based on such concepts may vary from one time-point to another in the semantics. This is the view that would be sanctioned by a non-nativist approach. However, certain models of the semantics allow for a uniform or constant formation of sortal concepts and, consequently, for a constant set of knowable propositions. Although this does not imply necessarily a nativist view of concept-formation, this view can philosophically ground such models. In other words, if we assume radical nativism, those models might be interpreted as representing this view as far as the concept of knowability is concerned. Partial nativism is not being taken into account.

3 Language and Semantics

We proceed now to caracterize the formal language of this paper. The set of its primitive logical symbols will be constituted by the expressions $\neg, \rightarrow, =, \forall$, (,), G, H and $[K]$. The first two symbols will stand for classical negation and material implication, the penultimate and antepenultimate symbols will formally represent the "it will always be the case" and the "it has always been the case" temporal propositional operators, and the last symbol the epistemic operator "it is known in principle that p relative to the set of sortal concepts that have been developed". The classical propositional operators of conjunction, disjunction and material equivalence will be represented by the symbols &, $\vee$ and $\leftrightarrow$, respectively, and defined in the usual way.

We shall assume denumerably many individual variables, sortal term variables and, for each positive integer n, n-place predicate variables. We shall use "x", "y" and "z" with or without numerical subscripts to refer (in the metalanguage) to individual variables and upper case letters in italics (except for "P", "F", "G", "H" and "K") to refer to sortal term variables. Atomic well formed formulas are expressions either of the form of a relative identity ($a =_L b$), where a and b are individual variables and L is a sortal term variable, or of the form $\pi x_1 \dots x_n$, where π is an n-place predicate variable and $x_1, \dots, x_n$ are individual variables. The set of well formed formulas (*wffs*, for short) is the smallest set containing the atomic well formed formulas and such that $\neg\varphi$, $(\varphi \rightarrow \delta)$, $\forall x L\varphi$, $G\varphi$, $H\varphi$, $[K]\varphi$ and $\forall L\varphi$ are in the set whenever φ, δ are in the set, and L is a sortal term variable.

Intuitively speaking, the expressions "$x =_L y$" will represent sortal identity. For example, if L stands for the sortal concept of being a tiger, then *x is the same tiger as y* will be represented by the expression "$x =_L y$". Expressions of the form "$\forall x S\varphi$", for any given sortal term S, will formally represent a relative sortal quantification with respect to the sortal concept L. Thus, when S stands for the concept of being a horse, and π for the concept of being black, the expression "every horse is black" will be represented by "$\forall x S\pi x$". Finally, "$\forall L\varphi$" will represent universal (second-order) quantification over sortal concepts. That is, it will stand for expressions like "every sortal concept is such that φ".

Hereafter, we shall make use of lower case greek letters φ, σ, δ, ψ and γ to refer to wffs, π to refer to predicate variables and upper case greek letters such as Γ, Δ and Σ to refer to sets of wffs. We shall generally drop the use of parentheses in a given context, if ambiguity is not possible in that context. The concepts of a bound and free occurrence of a variable are understood in the usual way. If α and β are variables of the same type, then by $\varphi^{\alpha}/\ \beta$ is meant the wff that results by replacing each free occurrence of β by a free occurrence of α, if such a wff exists; otherwise $\varphi^{\alpha}/\ \beta$ is φ itself. We shall say that α is free for β in φ, if $\varphi^{\alpha}/\ \beta$ is not φ unless α is β.

We shall now proceed to characterize the semantic system for the above language. We begin by defining a *frame for an epistemic-tense language for sortals predicates* (*ETS*-frame, for short) as a structure $\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_t \rangle_{t \in \mathcal{T}}$, where

1. $\mathcal{D}$ is a domain of discourse, empty or otherwise.

2. $\mathcal{W}$ and $\mathcal{T}$ are non-empty sets.

3. $\mathcal{S}$ is a function from $\mathcal{W} \times \mathcal{T}$ into $\wp(\wp(\mathcal{D})^{\mathcal{W} \times \mathcal{T}})$ (where $\wp(\mathcal{D})$ is the power set of $\mathcal{D}$ and $\wp(\mathcal{D})^{\mathcal{W} \times \mathcal{T}}$ is the set of functions from $\mathcal{W} \times \mathcal{T}$ into $\wp(\mathcal{D})$).

4. $\mathcal{R}$ is a serially ordered relation in $\mathcal{T}$, i.e., $R \subseteq \mathcal{T} \times \mathcal{T}$, and $\mathcal{R}$ is transitive, irreflexive and connected. In other words, $\mathcal{R}$ satisfies the following conditions: (i) for every α, β, $\gamma \in \mathcal{T}$, if $\alpha \mathcal{R}\ \beta$ and $\beta \mathcal{R}\ \gamma$, then $\alpha \mathcal{R}\ \gamma$; (ii) for every α, β, $\in \mathcal{T}$, either $\alpha = \beta$ or $\alpha \mathcal{R}\ \beta$ or $\beta \mathcal{R}\ \alpha$; and for every $\alpha \in \mathcal{T}$, it is not the case that $\alpha \mathcal{R} \alpha$.

5. for each $t \in \mathcal{T}$, $\mathcal{K}_t$ is an equivalence relation in a subset of $\mathcal{W}$, that is, $\mathcal{K}_t \subseteq \mathcal{W} \times \mathcal{W}$ and $\mathcal{K}_t$ is symmetric, reflexive and transitive.

We should note that $\mathcal{D}$ represents the set of objects existing at some possible world or other, $\mathcal{W}$ the set of (epistemically accessible) possible worlds, $\mathcal{T}$ the set of time-points, and $\mathcal{R}$ the earlier-than relation.

For each t and w, $\mathcal{S}(\sqcup, \sqsupseteq)$ stands for the set of sortal concepts that have been constructed at a possible world w at time t (and maybe previously to t). Each function f that is a member of $\mathcal{S}(\sqcup, \sqsupseteq)$ set-theoretically represents a sortal concept C. Each one of such functions are to be intuitively understood as assigning to each possible world j and time-point k the set of objects existing at j and k that fall under the sortal concept C. This expresses the way sortal concepts are understood in the philosophical framework of this paper. According to this view, sortal concepts are cognitive capacities providing identity criteria only for things that exist.

Now, regarding time-points, that is, the members of T, we should note that a common sense view is assumed according to which time is serially ordered by the earlier-than relation. This relation is represented by R in the *ETS*-frame and its clause 4 gathers the common sense view on the ordering of time we have referred to.

Clause 5 represents the relativity of the notion of knowability to the set of concepts developed at a certain time point. The worlds accesible to a time-point will not be epistemically isolated from each other. They conform the same epistemic matrix based on the set of concepts developed at a time-point. For this reason, for every $t \in \mathcal{T}$, $\mathcal{K}_t$ is an equivalence relation.

By an *assignment* (of values to variables) in an *ETS*-frame $\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_{\ t}\rangle_{\ t\in\mathcal{T}}$, we shall understand a function $\mathcal{A}$ with the set of variables (of all types) as domain and such that (1) $\mathcal{A}(x) \in \mathcal{D}$, for each individual variable x, (2) $\mathcal{A}(L) \in \cup_{(i,j)\in\mathcal{W}\times\mathcal{T}}\mathcal{S}(i,j)$, for each sortal term variable L, and (3) for each positive integer n and n-place predicate variable π, $\mathcal{A}(\pi) \in \wp(\mathcal{D})^{\mathcal{W}\times\mathcal{T}}$. Clause 2 states that no concept exists other than those formed in a possible world. This corresponds to the nature of concepts as cognitive capacities or structures based on such capacities. Concepts ontologically depend on the concrete individuals existing at one world or other, and so there cannot be concepts other than those formed at a possible world.

By an *epistemic-tense sortal model* (*ETS*-model, for short) we shall mean an ordered pair $\mathfrak{A} = \langle\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_{\ t}\rangle_{t\in\mathcal{T}}, \mathcal{A}\rangle$, where $\mathcal{A}$ is an assignment in the *ETS*-frame $\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_{\ t}\rangle_{\ t\in\mathcal{T}}$. If $\mathfrak{A}$ is $\langle\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_{\ t}\rangle_{\ t\in\mathcal{T}}, \mathcal{A}\rangle$, by $\mathfrak{A}(d/a)$ should be understood the ordered pair $\langle\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_{\ t}\rangle_{\ t\in\mathcal{T}}, \mathcal{A}(d,a)\rangle$, where $\mathcal{A}(d,a)$ is like $\mathcal{A}$ except for assigning d to a, and a is either an individual or sortal term variable.

Let $\mathfrak{A}$ be a *ETS*-model $\langle\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_t\rangle_{t\in\mathcal{T}}, \mathcal{A}\rangle$. Where $i \in \mathcal{W}$, $j \in \mathcal{T}$, we shall define *the truth-value of* φ *in* $\mathfrak{A}$ *at* i and j (in symbols, $Val(\varphi, \mathfrak{A}, i, j)$) as follows:

1. $Val(x =_L y, \mathfrak{A}, i, j) = 1$ if $\mathcal{A}(x) = \mathcal{A}(y)$ and $\mathcal{A}(y) \in \mathcal{A}(L)(i,j)$; otherwise $Val(x =_S y, \mathfrak{A}, i, j) = 0$.

2. $Val(\pi x_1 \ldots x_n, \mathfrak{A}, i, j) = 1$ if $\langle \mathcal{A}(x_1), \ldots, \mathcal{A}(x_n)\rangle \in \mathcal{A}(\pi)(< i, j >)$; otherwise $Val(\pi x_1 \ldots x_n, \mathfrak{A}, i, j) = 0$.

3. $Val(\neg\varphi, \mathfrak{A}, i, j) = 1$ if $Val(\varphi, \mathfrak{A}, i, j) \neq 1$; otherwise $Val(\neg\varphi, \mathfrak{A}, i, j) = 0$.

4. $Val(\varphi \rightarrow \gamma, \mathfrak{A}, i, j) = 1$ if $Val(\neg\varphi, \mathfrak{A}, i, j) = 1$ or $Val(\gamma, \mathfrak{A}, i, j) = 1$; otherwise $Val(\varphi \rightarrow \gamma, \mathfrak{A}, i, j) = 0$.

5. $Val(\forall L\varphi, \mathfrak{A}, i, j) = 1$ if for every $d \in \mathcal{S}(i,j)$, $Val(\varphi, \mathfrak{A}(d/L), i, j) = 1$; otherwise $Val(\forall S\varphi, \mathfrak{A}, i, j) = 0$.

6. $Val(\forall xL\varphi, \mathfrak{A}, i, j) = 1$ if for every $d \in \mathcal{A}(L)(i,j)$, $Val(\varphi, \mathfrak{A}(d/x), i, j) = 1$; otherwise $Val(\forall xS\varphi, \mathfrak{A}, i, j) = 0$.

7. $Val(K[\varphi], \mathfrak{A}, i, j) = 1$ if for every $k \in \mathcal{W}$, if $i\mathcal{K}_j k, Val(\varphi, \mathfrak{A}, k, j) = 1$; otherwise $Val([K]\varphi, \mathfrak{A}, i, j) = 0$.

8. $Val(G\varphi, \mathfrak{A}, i, j) = 1$ if for every $k \in \mathcal{T}$, if jRk, $Val(\varphi, \mathfrak{A}, \mathfrak{i}, k) = 1$; otherwise $Val(G\varphi, \mathfrak{A}, i, j) = 0$.

9. $Val(H\varphi, \mathfrak{A}, i, j) = 1$ if for every $k \in \mathcal{T}$, if kRj, $Val(\varphi, \mathfrak{A}, i, k) = 1$; otherwise $Val(H\varphi, \mathfrak{A}, i, j) = 0$.

Finally, a wff φ is said to be *ETS-valid* if and only if $Val(\varphi, \mathfrak{A}, i, j) = 1$ for any *ETS*-model $\mathfrak{A}$, possible world i and time-point j in $\mathfrak{A}$; and a set of wffs Γ is *ETS-satisfiable* if and only if there are an *ETS*-model $\mathfrak{B}$, possible world i and time-point j in $\mathfrak{B}$ such that for every $\varphi \in \Gamma$, $Val(\varphi, \mathfrak{B}, i, j) = 1$.

As the reader can verify, the present semantic system allows that an object (in the sense of a value of a free individual variable) may not be identifiable by any sortal concept at all, that is, $[K]\neg\exists S(x =_S x)$ is consistent in the semantic system. In other words, the semantics does not assume that every entity of the domain should fall under a sortal concept. Note also that, according to clause 6 above, first order (sortal) quantification at a given world is over objects that exist at that world (although the free individual variables have objects that exist at any world as values). This will have the consequence that the formal system characterized in the present paper will constitute a free logic regarding individual terms. By clause 5, such a formal system will also be a free logic with respect to sortal terms.

It is important to point out that the semantic system preserves Leibniz's law under relative (sortal) identity. Several strong arguments in favor of Leibniz's law for relative identity have been formulated, especially in Wiggins (2001) and Stevenson (1972)). These arguments justify our assuming the law in question for our semantics [3]. Now, semantic validation of Leibniz' law does not mean that the semantics does not allow for contingent identities. On the contrary, since we are assuming an approach to proper names according to which those names constitute sortal terms standing for (sortal) concepts that provide identity criteria for uniquely identifying (at most) one thing.

Now, as stated above, we are here taking into account three possible views regarding the influence of the environment in concept-formation. These are the radical, partial nativist and the non-nativist theories. *ETS*-models might be viewed as representing a non-nativist approach to the epistemic operator $[K]$. That is, the class of *ETS*-models might be interpreted as formally portraying a non-nativist approach regarding the epistemic operator $[K]$. Together both clauses 5 and 3 of an *ETS* frame convey this idea.

Now, the above semantics can allow for a formal representation of radical nativism. This is because it includes *ETS*-models that might represent a nativist approach to concept-development. These will be the models in which the set of concepts at a possible world of a *ETS* model does not vary through the different time-points of the same model. In formal terms, if $\mathfrak{A}$ be a *ETS*-model $\langle\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_t\rangle_{t\in\mathcal{T}}, \mathcal{A}\rangle$, for $i, m \in \mathcal{W}$, $j, k \in \mathcal{T}$, if $(i, m) \in \mathcal{K}_t$, then $\mathcal{S}(\ i, t) = \mathcal{S}(\ i, j)$. In addition to this condition, in such models, $\mathcal{K}_t = \mathcal{K}_m$, for every $t, m \in \mathcal{T}$. The set of knowable propositions does not vary through time, because we have the same set of sortal concepts at any time-point.

An *ETS*-model that fulfills the above two conditions will be here called a Nativist *ETS* model (*NETS*-model). On this basis, we can define the following notion of logical validity:

A wff φ is said to be *NETS-valid* if and only if $Val(\varphi, \mathfrak{A}, i, j) = 1$ for any *NETS*-model $\mathfrak{A}$, possible world i and time-point j in $\mathfrak{A}$; and a set of wffs Γ is *NETS-satisfiable* if and only if there are a *NETS*-model $\mathfrak{B}$, possible world i and time-point j in $\mathfrak{B}$ such that for every $\varphi \in \Gamma$, $Val(\varphi, \mathfrak{B}, i, j) = 1$.

In section 5, we shall prove the completeness and soundness of *NETS*-validity. Completeness and soundness for *ETS*-validity is left as an open problem.

4 The Formal System NETS

We proceed now to formulate a tense-modal formal logical system for sortals. We shall prove that this formal system is complete and sound with respect to *NETS*-validity, that is, with respect to the notion of logical validity provided by nativist epistemic-tense semantics for sortal predicates. For this reason, we shall refer to the formal system in question as NETS. Before characterizing the formal system NETS, we shall need the following definitions and convention:

Definition 0:

1. $\langle K\rangle\varphi = \neg[K]\neg\varphi$
2. $P\varphi = \neg H\neg\varphi$
3. $F\varphi = \neg G\neg\varphi$
4. $[t]\varphi = G\varphi \;\&\; H\varphi \;\&\; \varphi$

Clearly, the operators $\langle K\rangle$, P and F here defined, correspond to epistemic, future and past-tense possibility, respectively. The operator $[t]$ represents temporal necessity.

In the next definition, we assume that two sortal concepts are the same if and only they are co-extensive at any possible world and time-point.

Definition 1: If L and M are sortal term variables, y and x individual variables, then $(L{=}M) = [t][K](\forall yM\exists xL(x =_L y) \;\&\forall yL\exists xM(x =_M y))$

Convention 0: If φ be a wff, then (a) $\boxdot\varphi$ will represent one of the wffs $G\varphi, H\varphi$ and $[K]\varphi$; and (b) $\boxdot^*\varphi$ will represent the wff $G\varphi$ if $\boxdot\varphi$ is $H\varphi$, the wfff $H\varphi$ if $\boxdot\varphi$ is $G\varphi$ and the wff $[K]\varphi$ if $\boxdot\varphi$ is $[K]\varphi$.

Convention 1: (a) By $\odot\varphi$ we shall mean $\neg\boxdot\neg\varphi$. So $\odot\varphi$ might represent one of the wffs $F\varphi, P\varphi$ and $\langle K\rangle\varphi$, depending clearly on what $\boxdot\varphi$ is.

We are ready now to present NETS.

Axioms of NETS

A0. All tautologies
A1. $\forall L \exists M(L = M)$
A2. $\forall xL \exists yL(y =_L x)$
A3. $\varphi \to \forall yL\varphi$, provided y does not occur free in φ
A4 $\varphi \to \forall L\varphi$, provided L does not occur free in φ
A5.$x =_L x \to \exists yL(y =_L x)$, where y is a variable other than x
A6.$\exists L(L{=}M) \to (\forall L\varphi \to \varphi^M/L)$, provided M is free for L in φ
A7. $x =_L y \to x =_L x$
A8. $\forall xL(\varphi \to \gamma) \to (\forall xL\varphi \to \forall xL\gamma)$
A9. $\forall L(\varphi \to \gamma) \to (\forall L\varphi \to \forall L\gamma)$
A10. $(\langle K\rangle(y =_L z) \vee F\langle K\rangle(y =_R z) \vee P\langle K\rangle(y =_A z)) \to [t][K](\exists xM(y =_M x) \to (y =_M z))$, where y is a variable other than x
A11. $x =_L y \to (\varphi \to \varphi^*)$, where φ^* is obtained from φ by replacing one or more free occurrences of x by free occurrences of y.
A12. $[K]\varphi \to \varphi$
A13. $\langle K\rangle\varphi \to [K]\langle K\rangle\varphi$
A14. $[K](\varphi \to \sigma) \to ([K]\varphi \to [K]\sigma)$.
A15. $\varphi \to GP\varphi$
A16. $\varphi \to HF\varphi$
A17. $P\varphi \to H(F\varphi \vee \varphi \vee P\varphi)$
A18. $F\varphi \to G(\varphi \vee P\varphi \vee F\varphi)$
A19. $G\varphi \to GG\varphi$
A20. $H\varphi \to HH\varphi$
A21 $G(\varphi \to \sigma) \to (G\varphi \to G\sigma)$
A22 $H(\varphi \to \sigma) \to (H\varphi \to H\sigma)$
A23 $F[K]\varphi \to [K]F\varphi$
A24 $P[K]\varphi \to [K]P\varphi$
A25 $\exists L(L{=}M) \to [K][t]\exists L(L{=}M)$
A26$[K]\varphi \to [t][K][t]\varphi$
(Note: Hereafter, we shall refer respectively to axioms A11, A14, A21, A22, A23 and 24 as axioms (LL), ($DIST[K]$),(DISTG), (DISTH)), and bridge axioms

Rules of NETS:

$\boxdot Gen$: from $\sigma \to \boxdot_1(\gamma_1 \to \ldots \to \boxdot_n(\gamma_n \to \boxdot\varphi)\ldots)$ infer $\sigma \to \boxdot_1(\gamma_1 \to \ldots \to \boxdot_n(\gamma_n \to \boxdot\forall u\varphi)\ldots)$, provided u does not occur free in $\sigma \to \boxdot_1(\gamma_1 \to \ldots \to \boxdot_n(\gamma_n \to \boxdot\forall u\varphi)\ldots)$; $\gamma_1 \ldots \gamma_n$are wffs, u is either an individual variable or a sortal term variable, and for $0 < i \leq n$, $\boxdot_i \in \{G$, H and $[K]\}$
$UG(s)$: from φ infer $\forall L\varphi$

UG: from φ infer $\forall xL\varphi$
MP: from φ and $\varphi \to \sigma$ infer σ
RG: from φ infer $G\varphi$
RH: from φ infer $H\varphi$
RN: from φ infer $[K]\varphi$
Irr: from $([K](\pi x \,\& H\neg\pi x) \to \varphi$, infer φ, provided π does not occur in φ

(Note: In the case of $\boxdot Gen$, when $n = 0$, it becomes "from $\sigma \to \boxdot\varphi$ infer $\sigma \to \boxdot\forall u\varphi$")

We shall say that a wff φ is a theorem of NETS (in symbols, $\vdash \varphi$) if and only if there are wffs $\gamma_1, \ldots, \gamma_n$ such that for every i $(1 \leq i \leq n)$, γ_i is either an axiom or follows from previous wffs in the sequence by one of the rules of NETS, and γ_n is φ. A wff φ is an NETS-*theorem of* Γ (in symbols, $\Gamma \vdash \varphi$) if and only if there are wffs $\psi_1, \ldots, \psi_n \in \Gamma$ such that $(\psi_1 \& \ldots \& \psi_n) \to \varphi$ is a theorem of NETS.

Convention 2: From now on, a proof requiring reasoning in accordance with classical propositional logic will be denoted by PL.

We now state several theorems that are essential for the completeness proof in section 5. We shall also briefly indicate how to prove them.

Theorems

T0. $\exists xS(x =_S y) \to (\forall xS\varphi \to \varphi^y/x)$, provided y is a variable other than x free for x in φ
(LL, PL, UG, A3, A8)
T1. $\forall yS\varphi \leftrightarrow \forall zS\varphi^z/y$, provided z is free for y in φ and does not occur free in φ
(T0, UG, A8, A2, A3)
T2. $\forall S\varphi \leftrightarrow \forall H\varphi^H/S$, provided H is free for S in φ and does not occur free in φ
(A6, $UG(s)$, A9, A4)
T3. $x =_S y \to y =_S x$
(LL, A7, PL)
T4. $x =_S y \to \exists zS(z =_S x)$
(A7, A5 and PL)
T5. $\exists yS(x =_S y) \to x =_S x$
(A7, PL, UG, A8, A3, definition)
T6. $[K]\varphi \to [K](\varphi \,\&\, \varphi)$
(PL, R$[K]$, Dist$-[K]$)
T7.$[K](\varphi \,\&\, \psi) \to [K](\psi \,\&\, \varphi)$
(PL, R$[K]$, Dist$-[K]$)

T8.$([K](\varphi \ \& \ \psi) \& [K](\psi \ \& \ \delta)) \to [K](\varphi \ \& \ \delta)$
(PL, R$[K]$, Dist$-[K]$)
T9.$\langle K \rangle(x =_S y) \to \langle K \rangle(y =_S x)$
(T3, R$[K]$, Dist$[K], PL, definition$)
T10.$F\langle K \rangle(x =_S y) \to F\langle K \rangle(y =_S x)$
(T9, PL, $RG, DistG, PL, definition$)
T11. $P\langle K \rangle(x =_S y) \to P\langle K \rangle(y =_S x)$
(T9, PL, $RH, DistH, PL, definition$)
T12.$(G[K]\varphi \to G[K](\varphi \ \& \ \varphi)$
(T6, PL, RG, Dist$G$)
T13.$(H[K]\varphi \to H[K](\varphi \ \& \ \varphi)$
(T6, PL, RH, Dist$H$)
T14. $([K]\varphi \& \langle K \rangle \gamma) \to \langle K \rangle(\varphi \& \gamma)$
(PL, R$[K]$, Dist$-[K], PL$)
T15 $(H\varphi \& P\gamma) \to P(\varphi \& \gamma)$
(PL, RG, DistG, PL)
T16. $(G\varphi \& F\gamma) \to F(\varphi \& \gamma)$
(PL, RH, DistH, PL)
T17.$(\langle K \rangle(x =_M y) \ \& \ \langle K \rangle(y =_L z)) \to \langle K \rangle(x =_L z)$
(A10, T9, T4, T6, T14, PL, R$[K]$, LL)
T18.$F([K]\varphi \& \langle K \rangle \gamma) \to F\langle K \rangle(\varphi \& \gamma)$
(T14, RG,Dist$G$, , PL)
T19.$P([K]\varphi \& \langle K \rangle \gamma) \to P\langle K \rangle(\varphi \& \gamma)$
(T14, RH,DistH, PL)
T20.$\langle K \rangle(x =_S y) \ \& \ F\langle K \rangle(y =_L z) \to F\langle K \rangle(x =_L z)$
(A10, T9, T4, T6, T16, PL, R$[K]$, RG, LL)
T21.$\langle K \rangle(x =_S y) \ \& \ P\langle K \rangle(y =_L z) \to P\langle K \rangle(x =_L z)$
(A10, T9, T4, T6, T15, PL, R$[K]$, RG, LL)
T22.$F\langle K \rangle(x =_S y) \ \& \ P\langle K \rangle(y =_L z) \to P\langle K \rangle \exists S(x =_L z)$
(A10, T9, T4, T15, PL, R$[K]$, RG, LL)
T23.$P\langle K \rangle(x =_S y) \ \& \ F\langle K \rangle \exists S(y =_L z) \to F\langle K \rangle(x =_S z)$
(A10, T9, T4, T16, PL, R$[K]$, RG, LL)
T24.$P\langle K \rangle(x =_S y) \ \& \ P\langle K \rangle(y =_L z) \to P\langle K \rangle(x =_L z)$
(A10, T9, T4,T6, T17, PL, R$[K]$, RH, LL)
T25.$F\langle K \rangle(x =_S y) \ \& \ F\langle K \rangle(y =_L z) \to F\langle K \rangle \exists S(x =_L z)$
(A10, T9, T4, T16, PL, R$[K]$, RG, LL)
T26. $[K]\varphi \to [K][K]\varphi$
(A12, A13, PL, and definitions)
T27. $P\langle K \rangle(x =_S y) \& \ \langle K \rangle(y =_L z) \to P\langle K \rangle \exists S(x =_L z)$
(A10, T9, T4, T6, T14, PL, R$[K]$,RH LL)

T28. $F\langle K\rangle(x =_S y)\& \langle K\rangle(y =_L z) \to F\langle K\rangle(x =_L z)$
(A10, T9, T4, T6, T14, PL, R$[K]$, RG, LL)
T29. $([t][K](\varphi \ \& \ \psi)\& \ [t][K](\psi \ \& \ \delta)) \to [t][K](\varphi \ \& \ \delta)$
(T8, RG, RH, Dist$G$, Dist$H$,PL)
T30. $([t][K](\varphi \ \& \ \psi)) \to [t][K](\psi \ \& \ \varphi)$
(T7, RG, RH, Dist$G$, Dist$H$,PL)
T31. $([K]\varphi \ \& \ [K]\psi) \to [K](\varphi \ \& \ \psi)$
T32. $[K]G\varphi \to G[K]\varphi$
(PL, A15, A24, T16, A16, T14)
T33. $G[K]\varphi \to [K]G\varphi$
(PL, A23, A13, RG, A12, RN)
T34. $[K]H\varphi \to H[K]\varphi$
(PL, A16, A23, T15, A15, T14)
T35. $H[K]\varphi \to [K]H\varphi$
(PL, A24, A13, RH, A12, RN)
T36.$\varphi \to [K]\langle K\rangle\varphi$
(PL, A12, A13)

Derived rule 1 (Replacement rule) : if $\vdash \varphi \leftrightarrow \psi$, then $\vdash \delta \leftrightarrow \delta^*$, where δ^* is the result of replacing one or more occurrences of φ by ψ in δ.
Proof. By strong induction on the complexity of δ. ■

Derived rule 2(a) if $\vdash \sigma \to \boxdot_1(\varphi_1 \to \ldots \boxdot_n (\varphi_n \to \boxdot_{n+1}(\exists x S(x =_S y) \to \psi)\ldots)$, then $\vdash \sigma \to \boxdot_1(\varphi_1 \to \ldots \boxdot_n (\varphi_n \to \boxdot_{n+1}\forall y\psi)\ldots)$, provided y is an individual variable that does not occur free in $\sigma \to \boxdot_1(\varphi_1 \to \ldots \boxdot_n (\varphi_n \to \boxdot_{n+1}\forall y\psi)\ldots)$ and for every $i \in \omega$, $\boxdot_{i+1} \in \{G, H, [K]\}$.
Proof. By $\boxdot Gen$, the fact that (by A2, A8, UG and PL) $\vdash \forall y(\exists x S(x =_S y) \to \psi) \leftrightarrow \forall y\psi$ and derived rule1. ■

Derived rule 2(b) if $\vdash \sigma \to \boxdot_1(\varphi_1 \to \ldots \boxdot_n (\varphi_n \to \boxdot_{n+1}(\exists L(L =_S M) \to \psi)\ldots)$, then $\vdash \sigma \to \boxdot_1(\varphi_1 \to \ldots \boxdot_n (\varphi_n \to \boxdot_{n+1}\forall M\psi)\ldots)$, provided M is a sortal term variable that does not occur free in $\sigma \to \boxdot_1(\varphi_1 \to \ldots \boxdot_n (\varphi_n \to \boxdot_{n+1}\forall M\psi)\ldots)$ and for every $i \in \omega$, $\boxdot_{i+1} \in \{G, H, [K]\}$.
Proof. Similar to the above proof. ■

Derived rule 3: If $\vdash \sigma \to \boxdot_0(\varphi_1 \to \ldots \boxdot_{n-1} (\varphi_n \to \boxdot_n\neg\psi)\ldots)$, then $\vdash \psi \to \boxdot^*_n(\varphi_n \to \ldots \boxdot^*_1 (\varphi_1 \to \boxdot^*_0\neg\sigma)\ldots)$ for every $i \in \omega$, $0 < i \leq n$, $\boxdot_i \in \{G,\ H$ and $[K]\}$.
Proof. Assume hypothesis. By A12-13, A15, A16, RN, RG, RH, Dist$G$, Dist$H$ and Dist$[K]$, it can be shown by weak induction that $\vdash \odot_{j-1}(\varphi_j \ \&.\odot_j(\varphi_{j+1} \ \& \ldots \& \odot_{n-1}(\varphi_n \ \& \ \odot_n\psi)\ldots) \to (\varphi_{j-1} \to \boxdot^*_{j-2}(\varphi_{j-2} \to \ldots \boxdot^*_1 (\varphi_1 \to \boxdot^*_0\neg\sigma)\ldots)$. Then by $j = n+1$,

$\vdash \odot_n\psi \to (\varphi_n \to \boxdot^*_{n-1}(\varphi_{n-1} \to \ldots \boxdot^*_1 (\varphi_1 \to \boxdot^*_0\neg\sigma)\ldots)$. By RN, RG or RH ; Dist$G$, Dist$H$ or Dist$[K]$, $\vdash \boxdot^*_n \odot_n\psi \to \boxdot^*_n(\varphi_n \to \boxdot^*_{n-1}(\varphi_{n-1} \to \ldots \boxdot^*_1 (\varphi_1 \to \boxdot^*_0\neg\sigma)\ldots)$ and so by A12-13,and A15-A16, $\vdash \psi \to \boxdot^*_n(\varphi_n \to \boxdot^*_{n-1}(\varphi_{n-1} \to \ldots \boxdot^*_1 (\varphi_1 \to \boxdot^*_0\neg\sigma)\ldots)$. ■

5 Completeness and soundness of NETS

We proceed to show that system NETS is sound and complete. First, we assume the following conventions and define certain notions, most of which are instrumental for the completeness proof.

Convention 3*:* Let φ be a wff. By $\lfloor\varphi\rfloor$ we shall mean any wff of the form $\odot_1(\gamma_1$ $\& \ldots \odot_{n-1} (\gamma_{n-1}$ $\&$ $\odot\varphi)\ldots)$, where $\gamma_1 \ldots \gamma_n$ are wffs, for $0 < i \leq n$, $\odot_i \in \{P, F$ and $\langle K\rangle\}$ and $\odot \in \{P, F$ and $\langle K\rangle\}$.(Note: when $n = 1$, then $\lfloor\varphi\rfloor$ is $\odot\varphi$).

Convention 4: If $\lfloor\varphi\rfloor$ is $\odot_1(\gamma_1$ $\& \ldots \odot_n$ $(\gamma_n$ $\&$ $\odot\varphi)\ldots)$, then in any given context in which a wff $\lfloor\varphi\rfloor$ occurs, then $\lfloor\psi\rfloor$ in the same context will be $\odot_1(\gamma_1$ $\& \ldots \odot_n$ $(\gamma_n$ $\&$ $\odot\psi)\ldots)$ unless otherwise stated

Definition 2
Let Γ be a set of wffs.

1. Γ is ω-complete if and only if Γ satisfies the following three clauses:(a) if $\exists xS\varphi \in \Gamma$, then there is a variable y other than x which is free for x in φ such that $(\exists xS(x =_S y)$ $\&$ $\varphi^y/x) \in \Gamma$;(b) for all wff φ, if $\exists S\varphi \in \Gamma$, then there is a sortal term T free for S in φ such that $(\exists S(S = T)$ $\&$ $\varphi^T/S) \in \Gamma$; and (c) for all wff φ, if $\lfloor\exists xS\varphi\rfloor \in \Gamma$, then there is a variable y other than x which is free for x in φ such that $\lfloor\exists xS(x =_S y)$ $\&$ $\varphi^y/x\rfloor \in \Gamma$

2. Γ is irreflexive if and only if both (a) for all wff φ, if $\lfloor\varphi\rfloor \in \Gamma$. then there is an one-place predicate variable R which does not occur in $\lfloor\varphi\rfloor$ and an individual variable x such that $\lfloor[K](Rx$ $\&$ $H\neg Rx)$ $\&$ $\varphi\rfloor \in \Gamma$; and (b) there is an one-place predicate variable R and individual variable x such that $[K](Rx$ $\&$ $H\neg Rx) \in \Gamma$

3. Γ is a NETS-*maximally consistent set* of wffs if and only if Γ is NETS-consistent and for every wff φ, either $\varphi \in \Gamma$ or $\neg\varphi \in \Gamma$.

4. Γ is a *maxc* set of wffs if and only if Γ is an irreflexive, maximally consistent ω-complete set of wffs.

Definition 3

Let φ be a wff, α an individual or sortal term variable. By recursion, we shall define the expression "$[\varphi^{(\alpha)}]$", which intuitively should be understood as the result of rewriting all bound occurrences of α by variables new to φ of the same type as α.

- If φ is an atomic wff, then $[\varphi^{(\alpha)}] = \varphi$
- If φ is of the form$\neg\psi$, then $[\varphi^{(\alpha)}] = \neg[\psi^{(\alpha)}]$
- If φ is of the form $\psi \to \gamma$, then $[\varphi^{(\alpha)}] = [\psi^{(\alpha)}] \to [\gamma^{(\alpha)}]$
- If φ is of the form $\forall zS\psi$, then
$$\begin{cases} [\varphi^{(\alpha)}] = \forall zS[\psi^{(\alpha)}], if\ z\ is\ not\ \alpha \\ [\varphi^{(\alpha)}] = \forall kS[\psi^{(\alpha)}]^k/\alpha, if\ z\ is\ \alpha \\ and\ k\ is\ the\ first\ individual\ variable\ new\ to\ both\ [\psi^{(\alpha)}]\ and\ \varphi. \end{cases}$$
- If φ is of the form $\forall S\psi$, then
$$\begin{cases} [\varphi^{(\alpha)}] = \forall S[\psi^{(\alpha)}], if\ S\ is\ not\ \alpha \\ [\varphi^{(\alpha)}] = \forall H[\psi^{(\alpha)}]^H/\alpha,\ if\ S\ is\ \alpha \\ and\ H\ is\ the\ first\ sortal\ term\ variable\ new\ to\ both\ [\psi^{(\alpha)}]\ and\ \varphi \end{cases}$$
- If φ is of the form $[K]\psi$, then $[\varphi^{(\alpha)}] = [K][\psi^{(\alpha)}]$

Definition 4:For every maxc $\Gamma, \Sigma, \Gamma R_{\boxdot}\Sigma$ if and only if for every wff φ, if $\boxdot\varphi \in \Gamma$, then $\varphi \in \Sigma$ (where $\boxdot \in \{G, H, [K]\}$)

We shall now state and prove or indicate how to prove several lemmas indispensable for the completeness or soundness proofs.

Lemma 0: For any maxc Γ and Σ, and for any wff φ
I.The following are equivalents
(a) whenever $\varphi \in \Gamma$, we have $P\varphi \in \Sigma$
(b) whenever $\varphi \in \Sigma$, we have $F\varphi \in \Gamma$
(c) whenever $G\varphi \in \Gamma$, we have $\varphi \in \Sigma$
(d) whenever $H\varphi \in \Sigma$, we have $\varphi \in \Gamma$
II.The following are equivalents:
(e)whenever $[K]\varphi \in \Sigma$, we have $\varphi \in \Gamma$
(f) whenever $\varphi \in \Gamma$, we have $\langle K\rangle\varphi \in \Sigma$
Proof. By A15, A16, A12, A13 and PL. ∎

Lemma 1. $R_{[K]}$ is an equivalence relation in the set of maxc sets.

Proof. by A12, A13. ■

Definition 5: For every maxc set Γ, $[\Gamma]_{R_{[K]}}$ is the equivalence class of Γ determined by $R_{[K]}$ in the set of maxc sets.

Definition 6: For every maxc $\Gamma, \Sigma, \Gamma \simeq_G \Sigma$ if and only if $\Gamma = \Sigma$ or $\Gamma R_G \Sigma$ or $\Sigma R_G \Gamma$

Lemma 2. (a) R_G is transitive (b) the restriction of R_G to an arbitrary set of maxc sets of wffs is an irreflexive relation (c) R_G is left- and right-serial (d) $\simeq_G$ is an equivalence relation in the set of maxc sets.
Proof. (a) By axiom A19; (b) By A12, Lemma 0 and condition (b) of definition of irreflexive sets; (c) By axioms A17, A18, A19, A20 and (b) of this lemma; (d) By definition of $\simeq$, symmetry and reflexivity are obvious; its transitivity follows from (a) and (c) of this lemma. ■

Definition: Let $[\Gamma]_{\simeq_G}$ the equivalence class of Γ determined by $\simeq_G$ in the set of maxc consistent sets.

Lemma 3. The relation R_G is a serial order on every equivalence class $[\Gamma]_{\simeq_G}$
Proof. By lemma 2. ■

Lemma 4. *If* Γ and Σ are maxc sets of wffs such that $\Gamma \simeq_G \Sigma$ and there is a one-place predicate variable P and an individual variable x such that
$(Px \& H\neg Px) \in \Gamma \cap \Sigma$, then $\Gamma = \Sigma$.
Proof. By definitions of $\simeq_G$ and R_G, lemma 0 and the consistency of both Γ and Σ. ■

The following lemmas 5 and 6 can be easily proved by induction on the complexity of γ using T1 (for Lemma 5) and T2 (for Lemma 6).
Lemma 5 . For any individual variable x, $\vdash [\gamma^{(x)}] \leftrightarrow \gamma$

Lemma 6 .For any sortal term variable S, $\vdash [\gamma^{(S)}] \leftrightarrow \gamma$.

Note that : (i) If x is free for y in γ, then $[\gamma^x/y^{(x)}]$ is $[\gamma^{(x)}]^x/y$ and so by Lemma 5, $\vdash [\gamma^{(x)}]^x/y \leftrightarrow \gamma^x/y$; and (ii) If S is free for H in γ, then $[\gamma^S/H^{(S)}]$ is $[\gamma^{(S)}]^S/H$ and so by Lemma 6, $\vdash [\gamma^{(S)}]^S/H \leftrightarrow \gamma^S/H$

Lemma 7. For any wff φ, if $\mathfrak{A} = \langle\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_t\rangle_{t\in\mathcal{T}}, \mathcal{A}\rangle$, is a NETS-model and y is an individual variable free for x in φ, then for every $j \in \mathcal{W}, t \in \mathcal{T}, Val(\varphi, \mathfrak{A}(\mathcal{A}(y)/x), j, t) = 1$ if and only if $Val(\varphi^y/x, \mathfrak{A}, j, t) = 1$.
Proof. Let $C = \{n \in \omega|$ For any wff φ, if φ is of complexity n and $\mathfrak{A} = \langle\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_t\rangle_{t\in\mathcal{T}}, \mathcal{A}\rangle$ is a NETS-model and y is an individual variable free for x in φ, then for

every $j \in \mathcal{W}$, $t \in \mathcal{T}$, $Val(\varphi, \mathfrak{A}(\mathcal{A}(y)/x), j, t) = 1$ if and only if $Val(\varphi^y/x, \mathfrak{A}, j, t) = 1\}$. By strong induction it can be shown that $\omega \subseteq C$. ■

Lemma 8. For any wff φ, If $\mathfrak{A} = \langle\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_{\ t}\rangle_{t\in\mathcal{T}}, \mathcal{A}\rangle$ is a *NETS*-model and M is a sortal term variable free for L in φ, then for every $j \in \mathcal{W}$, $t \in \mathcal{T}$, $Val(\varphi, \mathfrak{A}(\mathcal{A}(L)/M), j, t) = 1$ if and only if $Val(\varphi^M/L, \mathfrak{A}, j, t) = 1$.

Proof for lemma 8 proceeds in a way analogous to the above proof for lemma 7.

Lemma 9: If φ is a wff, $\mathfrak{A} = \langle\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_{\ t}\rangle_{t\in\mathcal{T}}, \mathcal{A}\rangle$ and $\mathfrak{B} = \langle\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_{\ t}\rangle_{t\in\mathcal{T}}, \mathcal{B}\rangle$ are *NETS*-models such that $\mathcal{A}$ and $\mathcal{B}$ agree on all variables occurring free in φ, then $Val(\varphi, \mathfrak{A}, i, j) = 1$ if and only if $Val(\varphi, \mathfrak{B}, i, j) = 1$, for every $i \in \mathcal{W}$ and $j \in \mathcal{T}$.
Proof. By a straightforward induction on the complexity of sub-wffs of φ. ■

Metatheorem I (Soundness theorem): if φ is a theorem of NETS, then φ is *NETS*-valid
Proof. By induction on theorems. Directly from the semantic clauses, it can be shown the validity of axioms A1, A2, A5, A7-A10, A14-A16, A21-A26, and rules RG, RH, RN, UG, UG(s), $\boxdot$*Gen* and MP. The validity of A19-A20 and A17-18 follows from the transitivity and connectivity of the later-than relation, respectively, and the semantic clauses, and that of A12-13 follows from the assumption that the epistemic accessibility relation is reflexive and euclidean, respectively. For axioms A3 and A4, lemma 9 is needed, and for A6 lemma 8, in addition to the semantic clauses. For the case of Irr, assume first φ is not valid (where π does not occur in this latter formula). Therefore, there is a *NETS*-model $\mathfrak{A} = \langle\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, T, R, \mathcal{K}_t\rangle_{t\in\mathcal{T}}, \mathcal{A}\rangle$ and $w \in \mathcal{W}$, $t \in \mathcal{T}$, such that $Val(\varphi, \mathfrak{A}, w, t) = 0$. Let $\mathfrak{A}^* = \langle\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, T, R, \mathcal{K}_t\rangle_{t\in\mathcal{T}}, \mathcal{A}^*\rangle$, where $\mathcal{A}^*$ is like $\mathcal{A}$ except for what it assigns to the monadic predicate variable π. The function $\mathcal{A}^*$ assigns to π that function $f_\pi \in \wp(\mathcal{D})^{\mathcal{W}\times\mathcal{T}}$ such that, for every $(j, k) \in \mathcal{W} \times \mathcal{T}$,

$$f_\pi(j,k) = \begin{cases} A(\pi)((j,k)) \cup \{A(x)\}, \text{ if either } t = k \text{ or } tRk \\ \quad A(\pi)((j,k)) - \{A(x)\} \text{ otherwise} \end{cases}$$

Clearly, by lemma 9 and the irreflexivity of the R-relation among the members of $\mathcal{T}$ in $\mathfrak{A}^*$, $Val(([K](\pi x\ \& H\neg\pi x)) \to \varphi)), \mathfrak{A}^*, w, t) = 0$,and so $[K](\pi x\ \& H\neg\pi x) \to \varphi)$ is not valid. ■

Lemma 10 . If Γ is maxc and $\odot\gamma \in \Gamma$, then there is a maxc Σ such that $\gamma \in \Sigma$ and $\{\psi \mid \boxdot\psi \in \Gamma\} \subseteq \Sigma$, (where $\boxdot \in \{G, H$ and $[K]\}$).
Proof. Assume hypothesis. By *Convention* 0, "$\odot\gamma$" stands for $\neg \boxdot \neg\gamma$. Let $\delta_1, \ldots, \delta_n, \ldots$ be an ordering of wffs of the form either $\exists y S\varphi$, $\lfloor\varphi\rfloor$ or $\exists S\varphi$ [4]. Recursively define a sequence of wffs $\psi_0, \ldots, \psi_n, \ldots$ as follows.

i) $\psi_0 = \gamma$

ii) If $\odot(\psi_0\& \ldots \&\psi_n\&\ \delta_{n+1}) \notin \Gamma$, then $\psi_{n+1} = \psi_n$

iii) If $\odot(\psi_0\& \ldots \&\psi_n\&\ \delta_{n+1}) \in \Gamma$, then

iiia) if δ_{n+1} is of the form $\exists yS\varphi$,

$\psi_{n+1} = (\exists yS(y =_S x)\&\varphi^x/y))\}$

where x is the first variable other than y which is free for y in φ such that $\odot(\psi_0\ \& \ldots \&\ \psi_n\ \&\ (\exists yS(y =_S x)\ \&\ \varphi^x/y) \in \Gamma$, (*see footnote 5*)

iiib) if δ_{n+1} is of the form $\lfloor\varphi\rfloor$, then

$$\psi_{n+1} = \begin{cases} \lfloor[K](Rx\&H\neg Rx)\ \&\ \exists yS(y =_S z)\&\ \sigma^z/y)\rfloor, \text{ if } \varphi \text{ is } \exists yS\sigma, \text{ for some wff } \sigma; \\ \lfloor(\Box(Rx\&H\neg Rx)\ \&\ \exists S(S = T)\&\ \sigma^T/S)\rfloor, \text{ if } \varphi \text{ is } \exists S\sigma, \text{ for some wff } \sigma; \\ \lfloor[K](Rx\&H\neg Rx)\&\ \varphi\rfloor \text{ otherwise} \end{cases}$$

(where (1) both R is the first predicate variable and x the first individual variable which do not occur in $\odot(\psi_0\& \ldots \&\psi_n\&\ \delta_{n+1})$, if φ is not of the form $\exists yS\sigma$, for some wff σ; and (2) if φ is of the form $\exists yS\sigma$, for some wff σ, z is the first individual variable other than y which is free for y in σ such that $\odot(\psi_0\& \ldots \&\ \psi_n\&\ \lfloor\ \exists yS(y =_S z)\&\ \sigma^z/y)\rfloor \in \Gamma$ and R is the first predicate variable which do not occur in $\odot((\gamma_0\& \ldots \&\ \gamma_n)\&\ \lfloor\exists yS(y =_S z)\ \&\ \sigma^z/y)\rfloor)$ and x the first individual variable such that $\odot((\gamma_0\& \ldots \&\ \gamma_n)\&\ \lfloor[K](Rx\&H\neg Rx)\&\ \exists yS(y =_S z)\&\sigma^z/y\rfloor) \in \Gamma$. (*See footnote 5*).;(3) if φ is of the form $\exists S\sigma$ for some wff σ, T is the first sortal variable other than S which is free for S in σ such that $\odot(\psi_0\& \ldots \&\ \psi_n\&\ \lfloor\exists S(S = T)\&\ \sigma^T/S\rfloor) \in \Gamma$ and R is the first predicate variable which does not occur in $\odot((\gamma_0\& \ldots \&\ \gamma_n)\&\ \lfloor\exists S(S = T)\&\sigma^T/S\rfloor)$ and x the first individual variable such that $\odot((\gamma_0\& \ldots \&\ \gamma_n)\&\ \lfloor\Box(Rx\&H\neg Rx)\&\ (\exists S(S = T)\&\ \sigma^T/S\rfloor) \in \Gamma$.(*See footnote 5*).

iiic) if δ_{n+1} is of the form $\exists S\varphi$, then $\psi_{n+1} = (\exists S(S = L)\&\ \varphi^L/S)$ (where L is the first sortal term variable such that $\odot(\psi_0\& \ldots \&\ \psi_n\ \&\ (\varphi^L/S) \ldots) \in \Gamma$. (*see footnote 5*)

On the basis of the above recursion, it can be easily shown that for all $n \in \omega$, $\odot(\psi_0\& \ldots \&\ \psi_n) \in \Gamma$ and then that for all $n \in \omega$, $\{\psi_0\& \ldots \&\ \psi_n\}$ is consistent. Let $\Sigma = \{\varphi|\ \boxdot\varphi \in \Gamma\} \cup \{\psi_n : n \in \omega\}$. By *reductio ad absurdum*, we will show that Σ is consistent.

So suppose Σ is not consistent. Then there are n, $m \in \omega$ such that $\{\varphi_0, \ldots, \varphi_n, \psi_0, \ldots, \psi_m\} \subseteq \Sigma$ and $\vdash \neg(\varphi_0\& \ldots \&\ \varphi_n\&\ \psi_0\ \& \ldots \&\psi_m)$. So, by R$\boxdot$ and definitions, $\vdash \neg \odot (\varphi_0\& \ldots \&\ \varphi_n\&\ \psi_0\& \ldots \&\ \psi_n)$; then by maximality of $\Gamma, \neg \odot (\varphi_0\& \ldots \&\ \varphi_n\&\ \psi_0\& \ldots \&\ \psi_n) \in \Gamma$. On the other hand (since $\{\boxdot\varphi_0\& \ldots \&\ \boxdot\varphi_n\} \subseteq \Gamma$, Γ is maxc, and $\odot(\psi_0\& \ldots \&\ \psi_n) \in \Gamma$), by T14-16 $\odot(\varphi_0\& \ldots \&\ \varphi_n\&\ \psi_0\ \& \ldots \&\ \psi_n) \in \Gamma$, which is impossible by the consistency of Γ. Therefore, Σ is consistent.

We assume without loss of generality that there are one-place predicate variables not occurring in Σ. Otherwise for each $m \in \omega$, replace the *m-th* one-place predicate variable in all the wffs in Σ by the 2m-th one-place predicate variable. It can be easily shown that the replacement set for Σ is consistent if Σ is consistent. In the ordering of one-place predicate variables, let R be the first of the predicate variables not occurring in Σ. Let $K = \Sigma \cup \{[K](Rx\&H\neg Rx)\}$. By the Irr rule and PL, K is consistent. By Lindenbaum's method, extend K to a maximally consistent set K^*. Since $\{\psi_n : n \in \omega\} \subseteq K^*$, K^* is ω-complete. It is clearly irreflexive as well. Also by construction, $\gamma \in K^*$ and $\{\varphi \mid \boxdot\varphi \in \Gamma\} \subseteq K^*$. ■

Metatheorem II (Completeness Theorem for NETS): *If* Δ *is* NETS-*consistent*, *then* Δ *is* NETS-*satisfiable.*

Proof. Assume the hypothesis of the theorem. Without loss of generality, assume there are denumerably many individual variables $y_1, \ldots, y_n, \ldots$, denumerably many sortal term variables $L_0, \ldots, L_n \ldots$, and denumerably one-place predicate variables $R_0, \ldots, R_n \ldots$ which do not occur in Δ. (Otherwise for each $k, m, n \in \omega$, replace the k-*th* individual variable, the m-*th*-sortal term variable and n-*th* one-place predicate variable in all the wffs in Δ by the $2k$-th individual variable, the 2m-th sortal term variable and 2n-th one-place predicate variable, respectively. It can then be easily shown that Δ is satisfiable if and only if the replacement set for Δ is and that the replacement set for Δ is consistent if Δ is consistent). Let $\delta_0, \ldots, \delta_n \ldots$ be an enumeration of the wffs of the form $\exists yS\varphi$, $\lfloor\varphi\rfloor$, or $\exists S\varphi$. Let R^+ be the first predicate variable not occurring in Δ. By assumption and the Irr rule, $\Delta \cup \{[K](R^+x\&H\neg R^+x)\}$ is consistent. Define a chain of sets $\Gamma_0, \ldots, \Gamma_n, \ldots$ as follows.

1) $\Gamma_0 = \Delta \cup \{[K](R^+x\&H\neg R^+x)\}$

2) if δ_n is of the form $\exists yS\varphi$,

$\Gamma_{n+1} = \Gamma_n \cup \{(\exists yS\varphi \to (\exists yS(y =_S x)\&\varphi^x/y))\}$

(where x is the first individual variable new to $\Gamma_n \cup \{\delta_n\}$),

3) if δ_n is of the form $\lfloor\varphi\rfloor$, then

$$\Gamma_{n+1} = \begin{cases} \Gamma_n \cup \{\lfloor\varphi\rfloor \to \lfloor[K](Rx\&H\neg Rx)\&\ \exists yS(z =_S y)\&\ \sigma^z/y)\rfloor\}\ \text{, if } \varphi \text{ is} \\ \qquad \exists yS\sigma, \text{ for some wff } \sigma; \text{ or} \\ \Gamma_n \cup \{\lfloor\varphi\rfloor \to \lfloor[K](Rx\&H\neg Rx)\&\ \exists M(L =_M M)\&\ \sigma^L/M)\rfloor\}\ \text{, if } \varphi \text{ is} \\ \qquad \exists M\sigma, \text{ for some wff } \sigma; \\ \text{otherwise } \Gamma_n \cup \{\lfloor\varphi\rfloor \to \lfloor[K](Rx\&H\neg Rx)\ \&\ \varphi\rfloor\} \end{cases}$$

(where (a) both R is the first predicate variable and x is the first individual variable new to $\Gamma_n \cup \{\delta_n\}$, if φ is not of the form $\exists yS\sigma$ or $\exists S\sigma$ for some wff σ and, (b) if φ is $\exists yS\sigma$ for some wff σ, z is the first individual variable new to $\Gamma_n \cup \{\delta_n\}$), and both R is the first predicate variable and x is the first individual variable new to $\Gamma_n \cup \{\lfloor\varphi\rfloor \to \lfloor\exists yS(z =_S y)\&\ \sigma^z/y)\rfloor\}$; (c) if φ is $\exists M\sigma$ for some wff σ, L is the first sortal term variable new to $\Gamma_n \cup \{\delta_n\}$), and both R is the first predicate variable and x is the first individual variable new to $\Gamma_n \cup \{\lfloor\varphi\rfloor \to \lfloor\exists M(L =_M M)\&\ \sigma^L/M)\rfloor\}$

4) if δ_n is of the form $\exists M\varphi$,

$\Gamma_{n+1} = \Gamma_n \cup \{\exists M\varphi \to \exists M(L =_M M)\&\varphi^L/M\}$ (where L is the first sortal term variable new to $\Gamma_n \cup \{\delta_n\}$).

By weak induction, it can be shown that Γ_n is consistent, for every $n \in \omega$. Set $\Gamma^* = \bigcup_{n\in\omega} \Gamma_n$. Clearly, Γ^* is consistent. By Lindenbaum's method, extend Γ^* to a maximally consistent set Δ^*. Note that by construction Δ^* is ω-complete and irreflexive. So Δ^* is maxc.

Define now a relation among the set of individual variables as follows:

$x \cong z$ if and only if either for some sortal term variable M, $F\langle K\rangle(x =_M z) \in \Delta^*$ or $P\langle K\rangle(x =_M z) \in \Delta^*$ or $\langle K\rangle(x =_M z) \in \Delta^*$; or for every sortal term variable M, $[t][K]((\neg\exists yM(y =_M x)\&\neg\exists yM(y =_M z)) \in \Delta^*$.

Statement 0: $\cong$ *is an equivalence relation in the set of individual variables*

Proof:

1) $\cong$ is reflexive, *i.e.*, $x \cong x$, for every individual variable x: by Reductio Ad Absurdum, PL, T5,T14, T18 and T19.

2) $\cong$ is symmetric, that is, if $x \cong z$, then $z \cong x$, for every individual variable z and x: by T9, T10, T11, T30 and PL.

3) $\cong$ is transitive, *i.e.*, if $x \cong z$ and $z \cong w$, then $x \cong w$, for every individual variable x, z and w: by PL, T17, T20-T25, T27-8, T29, T14-16 and consistency of Δ^*. ▲

Let $[x]$ be the equivalence class of x determined by $\cong$ in the set of individual variables and set $\mathcal{D} = \{[x]|\ x$ is an individual variable$\}$. Define now a relation among the equivalence classes of *maxc* sets of wffs modulo $\simeq_G$ as follows: $[\Gamma]_{\simeq_G} \equiv [\Gamma']_{\simeq_G}$ if and only if there are Σ, Σ' such that $\Sigma \simeq_G \Gamma$ and $\Sigma' \simeq_G \Gamma'$ and $\Sigma R_{[K]} \Sigma'$.

Statement 1: $\equiv$ is an equivalence relation.

Proof: Clearly, by A12, A13, $\equiv$ is symmetric and reflexive. By A23-24, lemma 10, the irreflexivity of maxc sets, T31, lemma 4 and the fact that $R_{[K]}$ is an equivalence relation, it can be shown that $\equiv$ is transitive. ▲

Set $\sum = \bigcup\{[\Gamma]_{\simeq_G} \mid [\Gamma]_{\simeq_G} \equiv [\Delta^*]_{\simeq_G}\}$. Obviously, every equivalence class modulo $R_{[K]}$ or modulo $\simeq_G$ is a subset $\sum$ or is disjoint with $\sum$.

Statement 2: If A is an equivalence class modulo $\simeq_G$ and B an equivalence class modulo $R_{[K]}$ both of which are subsets of $\sum$, then there is exactly one maxc Ξ of wffs such that $\Xi \in A \cap B$.

Proof: Assume hypothesis. Then for some $\Phi, \Psi \in \sum$, $[\Phi]_{\simeq_G} = A$ and $[\Psi]_{R_{[K]}} = B$, $[\Phi]_{\simeq_G} \equiv [\Delta^*]_{\simeq_G}$; also, $[\Psi]_{\simeq_G} \equiv [\Delta^*]_{\simeq_G}$ (since $\Psi \in \sum$). Since $\equiv$ is symmetric and transitive, $[\Phi]_{\simeq_G} \equiv [\Psi]_{\simeq_G}$. Also, by the irreflexivity of Ψ, A12 and lemma 4, $[\Psi]_{R_{[K]}} \cap [\Psi]_{\simeq_G} = \Psi$. By A23, A24, lemma 10, the irreflexivity of maxc sets, T31, lemma 4, there is a unique maxc $\Xi \in [\Phi]_{\simeq_G}$ such that $\Psi R_{[K]} \Xi$, i.e., $\Xi \in [\Psi]_{R_{[K]}}$. Consequently there is a unique maxc Ξ such that $\Xi \in [\Phi]_{\simeq_G} \cap [\Psi]_{R_{[K]}}$. ▲

Convention 5: Given Statement 2, if A is an equivalence class modulo $\simeq_G$ and B an equivalence class modulo $R_{[K]}$ both of which are subsets of $\sum$, by the expression $\Sigma^*_{(A,B)}$ we shall denote the unique $\Gamma \in A \cap B$.

Set $\mathcal{T} = \{[\Gamma]_{R_{[K]}} \mid [\Gamma]_{R_{[K]}} \subseteq \sum\}$ and $\mathcal{W} = \{[\Gamma]_{\simeq_G} \mid [\Gamma]_{\simeq_G} \subseteq \sum\}$ and let

- For every sortal term variable M, $C_M = \{\langle((([\Gamma]_{\simeq_G}, [\Theta]_{R_{[K]}})), \{[x] \in \mathcal{D} \mid \exists y M(x =_M y) \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}\}\rangle \mid ([\Gamma]_{\simeq_G}, [\Theta]_{R_{[K]}}) \in \mathcal{W} \times \mathcal{T}$, and y is a variable other than $x\}$.
- $\mathcal{S} = \{\langle((([\Gamma]_{\simeq_G}, [\Theta]_{R_{[K]}}), \{C_M \mid M$ is a sortal term variable such that $\exists L(L = M) \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$, provided L is a sortal term variable other than $F\}\rangle \mid ([\Gamma]_{\simeq_G}, [\Theta]_{R_{[K]}}) \in \mathcal{W} \times \mathcal{T}\}$.
- For every n-place predicate variable π, $\mathcal{D}_\pi = \{\langle\langle(([\Gamma]_{\simeq_G}, [\Theta]_{R_{[K]}})\rangle, \{\langle[x_1]\dots[x_n]\rangle \in \mathcal{D}^n \mid \pi x_1 \dots x_n \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}\}\rangle \mid ([\Gamma]_{\simeq_G}, [\Theta]_{R_{[K]}}) \in \mathcal{W} \times \mathcal{T}\}$.
- $\mathcal{R} = \{([\Theta]_{R_{[K]}}, [\Psi]_{R_{[K]}}) \in \mathcal{T} \times \mathcal{T} \mid$ there is a $[\Gamma]_{\simeq_G} \in \mathcal{W}$ such that $\Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})} R_G \Sigma^*_{([\Gamma]_{\simeq_G},[\Psi]_{R_{[K]}})}\}$.
- For every $[\Gamma]_{R_{[K]}} \in \mathcal{T}$, $\mathcal{K}_{[\Gamma]_{R_{[K]}}} = \{([\Lambda]_{\simeq_G}, [\Theta]_{\simeq_G}) \in \mathcal{W} \times \mathcal{W} \mid$ if $[K]\gamma \in \Sigma^*_{([\Lambda]_{\simeq_G},[\Gamma]_{R_{[K]}})}$, then $\gamma \in \Sigma^*_{([\Theta]_{\simeq_G},[\Gamma]_{R_{[K]}})}\}$.
- $\mathcal{A}$ is the function whose domain is the set of variables such that $\mathcal{A}(x) = [x]$, $\mathcal{A}(\pi) = \mathcal{D}_\pi$ and $\mathcal{A}(M) = C_M$.
- $\mathfrak{A}^* = \langle\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_t\rangle_{t \in \mathcal{T}}, \mathcal{A}\rangle$.

  Clearly, $\langle \mathcal{D}, \mathcal{S}, \mathcal{W}, \mathcal{T}, \mathcal{R}, \mathcal{K}_t\rangle_{t \in \mathcal{T}}$ is a *ETS*-frame. By A25-26, $\mathcal{K}_t$ (for every $t \in \mathcal{T}$) and $\mathcal{A}$ fulfill the two conditions for nativist models. Therefore, $\mathfrak{A}^*$ is a *NETS*-model.

Where $\mathfrak{A}^*$ is the above defined *NETS*-model, note that by Lemmas 5-8 and the soundness theorem, the following statements 3 and 4 can be easily shown:

Statement 3: For any wff φ, individual variables y and x, and $\Gamma \in \mathcal{W}$, $\Theta \in \mathcal{T}$, $Val(\varphi, \mathfrak{A}^*([x]/y), \Gamma, \Theta) = 1$ if and only if $Val([\varphi^{(x)}]^x/y, \mathfrak{A}^*, \Gamma, \Theta) = 1$.

Statement 4: For any wff φ, sortal term variables L and S and $\Gamma \in \mathcal{W}$, $\Theta \in \mathcal{T}$, $Val(\varphi, \mathfrak{A}^*(C_L/S), \Gamma, \Theta) = 1$ if and only if $Val([\varphi^{(L)}]^L/S, \mathfrak{A}^*, \Gamma, \Theta) = 1$.

Statement 5: For any $[\Theta]_{R_{[K]}}, [\Psi]_{R_{[K]}} \in \mathcal{T}$, and $[\Gamma]_{\simeq_G}$, $[K]_{\simeq_G} \in \mathcal{W}$,

$$\text{if } \Sigma^*_{([K]_{\simeq_G},[\Theta]_{R_{[K]}})} R_G \Sigma^*_{([K]_{\simeq_G},[\Psi]_{R_{[K]}})}, \text{ then } \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})} R_G \Sigma^*_{([\Gamma]_{\simeq_G},[\Psi]_{R_{[K]}})}.$$

Proof: Assume hypothesis. Since $\Sigma^*_{([K]_{\simeq_G},[\Psi]_{R_{[K]}})}$ is irreflexive, there is a monadic predicate variable Q such that $[K](Qx\&H\neg Qx) \in \Sigma^*_{([K]_{\simeq_G},[\Psi]_{R_{[K]}})}$ and so

$$F[K](Qx\&H\neg Qx) \in \Sigma^*_{([K]_{\simeq_G},[\Theta]_{R_{[K]}})}.$$

Then by A23, $[K]F(Qx\&H\neg Qx) \in \Sigma^*_{([K]_{\simeq_G},[\Theta]_{R_{[K]}})}$ which means by A12 and definitions that $F((Qx\&H\neg Qx) \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$. Therefore, by lemma 10 and 4, there is a unique maxc Ω such that $\Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}R_G\Omega$ and $(Qx\&H\neg Qx) \in \Omega$. Now, suppose $[K]\varphi \in \Sigma^*_{([K]_{\simeq_G},[\Psi]_{R_{[K]}})}$. By T31 $[K](\varphi\ \&\ (Qx\ \&\ H\neg Qx) \in \Sigma^*_{([K]_{\simeq_G},[\Psi]_{R_{[K]}})}$, and by the hypothesis, $F[K](\varphi\ \&\ (Qx\&H\neg Qx) \in \Sigma^*_{([K]_{\simeq_G},[\Theta]_{R_{[K]}})}$, from which it follows by A23, $[K]F(\varphi\ \&\ (Qx\ \&\ H\neg Qx) \in \Sigma^*_{([K]_{\simeq_G},[\Theta]_{R_{[K]}})}$.By A12 and definitions,$F(\varphi\ \&$ $(Qx\&H\neg Qx) \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$ and so by lemma 10 and above $\varphi \in \Omega$.Consequently, $\varphi \in \Omega$ if $[K]\varphi \in \Sigma^*_{([K]_{\simeq_G},[\Psi]_{R_{[K]}})}$. Then, by A12, A13, $[K]\varphi \in \Omega$ if only if $[K]\varphi \in \Sigma^*_{([K]_{\simeq_G},[\Psi]_{R_{[K]}})}$. Clearly, $\Omega \in [\Psi]_{R_{[K]}} \cap [\Gamma]_{\simeq_G}$. By statement 2, $\Omega = \Sigma^*_{([\Gamma]_{\simeq_G},[\Psi]_{R_{[K]}})}$. ▲

Let $I = \{\iota \in \omega \mid$ if φ is of complexity ι, then for every $\Gamma, \Theta \in \sum$, $Val(\varphi, \mathfrak{A}^*,$ $[\Gamma]_{\simeq_G}, [\Theta]_{R_{[K]}}) = 1$ iff $\varphi \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}\}$. It can be shown by strong induction that $\omega \subseteq I$.

Statement 6: $\omega \subseteq I$

Proof: Suppose that φ is of complexity k, $\Gamma, \Theta \in \sum$ and for every $i < k$, $i \in I$. There are seven cases to consider. The cases where φ is of the form $\neg\gamma$ or $\gamma \to \sigma$ can be easily shown by the inductive hypothesis.

1. φ is of the form $x =_M y$: $Val(\varphi, \mathfrak{A}^*, [\Gamma]_{\simeq_G}, [\Theta]_{R_{[K]}}) = 1$ if and only if (by definition) $\mathcal{A}(x) = \mathcal{A}(y)$ and $\mathcal{A}(y) \in \mathcal{A}(M)([\Gamma]_{\simeq_G}, [\Theta]_{R_{[K]}})$ if and only if (by definition) $[x] = [y]$ and $\exists zM(z =_M y) \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$ if and only if (by definition) $\exists zM(z =_M$ $y) \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$; and for some sortal term variable B, $P\langle K\rangle(x =_B y) \in \Delta^*$ or $F\langle K\rangle(x =_B y) \in \Delta^*$ or $\langle K\rangle(x =_B y) \in \Delta^*$, or for every sortal term variable B, $[t][K](\neg\exists zB(z =_B x)\&\neg\exists zB(z =_B y)) \in \Delta^*$.

Now, suppose that $\exists zM(z =_M y) \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$ and for some sortal term variable B, $F\langle K\rangle(x =_B y) \in \Delta^*$ or $P\langle K\rangle(x =_B y) \in \Delta^*$or $\langle K\rangle(x =_B y) \in \Delta^*$. By A10, PL and the fact that R_G is a serial ordering in $[\Delta^*]_{\simeq_G}$, $[K]((\exists xM(y =_M x) \to (y =_M z)) \in$ $\Sigma^*_{([\Delta^*]_{\simeq_G},[\Theta]_{R_{[K]}})}$. But $\Sigma^*_{([\Delta^*]_{\simeq_G},[\Theta]_{R_{[K]}})} \in [\Theta]_{R_{[K]}}$ and so $[K](\exists xM(y =_M x) \to (y =_M z) \in$ $\Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$, from which it follows by A12 that $(\exists xM(y =_M x) \to (y =_M z)) \in$ $\Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$. But given that $\exists zM(z =_M y) \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$, by PL $(y =_M z)) \in$ $\Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$.

On the other hand, if for every sortal term variable M, $[t][K](\neg\exists zM(x =_M z) \&\neg\exists zM(y =_M z)) \in \Delta^*$, then $[K](\neg\exists zM(x =_M z) \&\neg\exists zM(y =_M z)) \in \Sigma^*_{([\Delta^*]_{\simeq_G},[\Theta]_{R_{[K]}})}$, since R_G is a serial ordering in $[\Delta^*]_{\simeq_G}$. Clearly, since $\Sigma^*_{([\Delta^*]_{\simeq_G},[\Theta]_{R_{[K]}})} \in [\Theta]_{R_{[K]}}$, $[K](\neg\exists zM(x =_M z) \&\neg\exists zM(y =_M z)) \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$ and by A12 $(\neg\exists zM(x =_M z) \&\neg\exists zM(y =_M z)) \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$, which is impossible because by assumption $\exists zM(y =_M z) \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$. Therefore, it is not the case that for every sortal term variable M, $[t][K](\neg\exists zM(x =_M z) \&\neg\exists zM(y =_M z)) \in \Delta^*$.

Assume now that $x =_M y \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$. Then $\exists zM(z =_M y) \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$ by T4; and by A12 and A13 $[K]\langle K\rangle(x =_M y) \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$. But $\Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})} \in [\Theta]_{R_{[K]}}$ and so $[K]\langle K\rangle(x =_M y) \in \Sigma^*_{([\Delta^*]_{\simeq_G},[\Theta]_{R_{[K]}})}$, which by A12 means that $\langle K\rangle(x =_M y) \in \Sigma^*_{([\Delta^*]_{\simeq_G},[\Theta]_{R_{[K]}})}$. Since R_G is a serial ordering in $[\Delta^*]_{\simeq_G}$ and by lemma 0, $F\langle K\rangle(x =_M y) \in \Delta^*$ or $P\langle K\rangle(x =_M y) \in \Delta^*$ or $\langle K\rangle(x =_M y) \in \Delta^*$. Therefore, for some sortal term M, $F\langle K\rangle(x =_M y) \in \Delta^*$ or $P\langle K\rangle(x =_M y) \in \Delta^*$ or $\langle K\rangle(x =_M y) \in \Delta^*$ and consequently, either for some sortal term M, $F\langle K\rangle(x =_M y) \in \Delta^*$ or $P\langle K\rangle(x =_M y) \in \Delta^*$ or $\langle K\rangle(x =_M y) \in \Delta^*$, or for every sortal some sortal term M $[t][K]((\neg\exists zM(x =_M z) \&\neg\exists zM(y =_M z))) \in \Delta^*$; and $\exists zM(z =_M y) \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$

2. φ is of the form $\pi x_1 \dots x_n$: inmediate from definitions.

3. φ is of the form $\forall yM\gamma$: $Val(\varphi, \mathfrak{A}^*, [\mathit{\Gamma}]_{\simeq_G}, [\mathit{\Theta}]_{R_{[K]}}) = 1$ if and only if (by definition) for every $d \in \mathcal{A}(M)([\mathit{\Gamma}]_{\simeq_G}, [\mathit{\Theta}]_{R_{[K]}})$, $Val(\gamma, \mathfrak{A}^*(d/y), [\mathit{\Gamma}]_{\simeq_G}, [\mathit{\Theta}]_{R_{[K]}}) = 1$ if and only if (by definition) for every individual variable x, if $[x] \in \mathcal{A}(M)([\mathit{\Gamma}]_{\simeq_G}, [\mathit{\Theta}]_{R_{[K]}})$, then $Val(\gamma, \mathfrak{A}^*([x]/y), [\mathit{\Gamma}]_{\simeq_G}, [\mathit{\Theta}]_{R_{[K]}}) = 1$ if and only if (by Statement 3) for every individual variable x, if $[x] \in \mathcal{A}(M)([\mathit{\Gamma}]_{\simeq_G}, [\mathit{\Theta}]_{R_{[K]}})$, then $Val([\gamma^{(x)}]^x/ y, \mathfrak{A}^*, [\mathit{\Gamma}]_{\simeq G}, [\mathit{\Theta}]_{R_{[K]}}) = 1$ if and only if (by definition) for every individual variable x, if $\exists zM(x =_M z) \in \Sigma^*_{([\mathit{\Gamma}]_{\simeq_G},[\mathit{\Theta}]_{R_{[K]}})}$ (where z is a variable other than x), then $Val([\gamma^{(x)}]^x/ y, \mathfrak{A}^*, [\mathit{\Gamma}]_{\simeq_G}, [\mathit{\Theta}]_{R_{[K]}}) = 1$ if and only if (by the inductive hypothesis) for every individual variable x, if $\exists zM(x =_M z) \in \Sigma^*_{([\mathit{\Gamma}]_{\simeq_G},[\mathit{\Theta}]_{R_{[K]}})}$, then $[\gamma^{(x)}]^x/y \in \Sigma^*_{([\mathit{\Gamma}]_{\simeq_G},[\mathit{\Theta}]_{R_{[K]}})}$ if and only if (by ω-completeness and maximality of $\Sigma^*_{([\mathit{\Gamma}]_{\simeq_G},[\mathit{\Theta}]_{R_{[K]}})}$,T0, T1, Lemma 5 and note (i) inmediately following Lemmas 5-6) if and only if $\forall yM\gamma \in \Sigma^*_{([\mathit{\Gamma}]_{\simeq_G},[\mathit{\Theta}]_{R_{[K]}})}$.

4. φ is of the form $\forall S\gamma$: $Val(\varphi, \mathfrak{A}^*, [\mathit{\Gamma}]_{\simeq_G}, [\mathit{\Theta}]_{R_{[K]}}) = 1$ if and only if (by definition) for every $C_F \in \mathcal{S}([\mathit{\Gamma}]_{\simeq_G}, [\mathit{\Theta}]_{R_{[K]}})$, $Val(\gamma, \mathfrak{A}^*(C_F/S), [\mathit{\Gamma}]_{\simeq_G}, [\mathit{\Theta}]_{R_{[K]}}) = 1$ if and only if (by Statement 4) for every $C_F \in \mathcal{S}([\mathit{\Gamma}]_{\simeq_G}, [\mathit{\Theta}]_{R_{[K]}})$, *Val* $([\gamma^{(F)}]^F/S, \mathfrak{A}^*, [\mathit{\Gamma}]_{\simeq_G}, [\mathit{\Theta}]_{R_{[K]}}) = 1$ if and only if (by definition) for every sortal term variable F and individual variables x, y, if $\exists L(L =_S F) \in \Sigma^*_{([\mathit{\Gamma}]_{\simeq_G},[\mathit{\Theta}]_{R_{[K]}})}$ (where L is a sortal term variable other than F), then *Val* $([\gamma^{(F)}]^F/S, \mathfrak{A}^*, [\mathit{\Gamma}]_{\simeq_G}, [\mathit{\Theta}]_{R_{[K]}}) = 1$ if and only if (by the inductive hypothesis) for every sortal term variable F, if $\exists L(L =_S F) \in \Sigma^*_{([\mathit{\Gamma}]_{\simeq_G},[\mathit{\Theta}]_{R_{[K]}})}$,

then, $[\gamma^{(F)}]^F/S \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$ if and only if (by ω-completeness and maximality of $\Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$, Lemma 6 and note (ii) immediately following Lemmas 5-6, A6 and T2) $\forall S\gamma \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$.

5. φ is of the form $[\ K\ ]\gamma$: $Val(\varphi, \mathfrak{A}^*, [\Gamma]_{\simeq_G}, [\Theta]_{R_{[K]}}) = 1$ if and only if (by definition) for every $[M]_{\simeq_G} \in \mathcal{W}$, if $[\Gamma]_{\simeq_G} \mathcal{K}_{[\Theta]_{R_{[K]}}} [M]_{\simeq_G}, Val(\gamma, \mathfrak{A}^*, [M]_{\simeq_G}, [\Theta]_{R_{[K]}}) = 1$ if and only if (by the inductive hypothesis)for every $[M]_{\simeq_G} \in \mathcal{W}$, if $[\Gamma]_{\simeq_G} \mathcal{K}_{[\Theta]_{R_{[K]}}} [M]_{\simeq_G}$, $\gamma \in \Sigma^*_{([M]_{\simeq_G},[\Theta]_{R_{[K]}})}$.

Now, if $[K]\gamma \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$, then (since $\Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})} \in [\Theta]_{R_{[K]}}$ and by A12) $\gamma \in \Sigma^*_{([M]_{\simeq_G},[\Theta]_{R_{[K]}})}$ (for every$[M]_{\simeq_G} \in \mathcal{W}$ such that $[\Gamma]_{\simeq_G} \mathcal{K}_{[\Theta]_{R_{[K]}}} [M]_{\simeq_G}$). Suppose now that $[K]\gamma \notin \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$. Then, by Lemma 10, there is a *maxc* Λ such that $\neg\gamma \in \Lambda$ and $\{\psi \mid [K]\psi \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}\} \subseteq \Lambda$. Clearly, $\Lambda \in [\Theta]_{R_{[K]}}$ and then $\Lambda \in \sum$, which means that for some $\Psi \in \sum$, $\Lambda \in [\Psi]_{\simeq_G}$and so, by Statement 2, $\Lambda = \Sigma^*_{([\Psi]_{\simeq_G},[\Theta]_{R_{[K]}})}$. By construction, $[\Gamma]_{\simeq_G} \mathcal{K}_{[\Theta]_{R_{[K]}}} [\Psi]_{\simeq_G}$ Therefore, there is $[\Psi]_{\simeq_G} \in \mathcal{W}$ such that both $[\Gamma]_{\simeq_G} \mathcal{K}_{[\Theta]_{R_{[K]}}} [\Psi]_{\simeq_G}$and $\neg\gamma \in \Sigma^*_{([\Psi]_{\simeq_G},[\Theta]_{R_{[K]}})}$.

6. φ is of the form $G\gamma$: $Val(\varphi, \mathfrak{A}\ , [\Gamma]_{\simeq_G}, [\Theta]_{R_{[K]}}) = 1$ if and only if (by definition) for every $[K]_{R_{[K]}} \in \mathcal{T}$, if $[\Theta]_{R_{[K]}} \mathcal{R}\ [K]_{R_{[K]}}$, then $Val(\gamma, \mathfrak{A}\ , [\Gamma]_{\simeq_G}, [K]_{R_{[K]}}) = 1$ if and only if (by the inductive hypothesis) for every $[K]_{R_{[K]}} \in \mathcal{T}$, if $[\Theta]_{R_{[K]}} \mathcal{R}\ [K]_{R_{[K]}}$, then $\gamma \in \Sigma^*_{([\Gamma]_{\simeq_G},[K]_{R_{[K]}})}$.

Now, suppose $G\gamma \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$, and $[\Theta]_{R_{[K]}} \mathcal{R}\ [K]_{R_{[K]}}$. Then, by definition of $\mathcal{R}$, there is $[\Lambda]_{\simeq_G} \in \mathcal{W}$ such $\Sigma^*_{([\Lambda]_{\simeq_G},[\Theta]_{R_{[K]}})} R_G \Sigma^*_{([\Lambda]_{\simeq_G},[K]_{R_{[K]}})}$, which implies by statement 5 that $\Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})} R_G \Sigma^*_{([\Gamma]_{\simeq_G},[K]_{R_{[K]}})}$ and so, by assumption, that $\gamma \in \Sigma^*_{([\Gamma]_{\simeq_G},[K]_{R_{[K]}})}$. Suppose now that $G\gamma \notin \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$. Then, by Lemma 10, there is a *maxc* Λ such that $\neg\gamma \in \Lambda$ and $\{\psi \mid G\psi \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}\} \subseteq \Lambda$. Clearly, $\Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})} R_G \Lambda$ and so $\Lambda \in [\Gamma]_{\simeq_G}$. Then, $\Lambda \in \sum$, which means that for some $\Psi \in \sum, \Lambda \in [\Psi]_{R_{[K]}}$. By Statement 2, $\Lambda = \Sigma^*_{([\Gamma]_{\simeq_G},[\Psi]_{R_{[K]}})}$ and therefore, for some $[\Psi]_{R_{[K]}} \in \mathcal{T}$, $[\Theta]_{R_{[K]}} \mathcal{R}\ [\Psi]_{R_{[K]}}$ and $\neg\gamma \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Psi]_{R_{[K]}})}$.

7. φ is of the form $H\gamma$: since it proceeds along similar lines, proof for this clause can be easily constructed just by following the one for case 6. ▲

We have shown above that for every wff φ and $\Gamma, \Theta \in \sum$, $Val(\varphi, \mathfrak{A}^*, [\Gamma]_{\simeq_G}, [\Theta]_{R_{[K]}}) = 1$ iff $\varphi \in \Sigma^*_{([\Gamma]_{\simeq_G},[\Theta]_{R_{[K]}})}$, in particular, for every wff φ, $Val(\varphi, \mathfrak{A}^*, [\Delta^*]_{\simeq_G}, [\Delta^*]_{R_{[K]}}) = 1$ iff $\varphi \in \Sigma^*_{([\Delta^*]_{\simeq_G},[\Delta^*]_{R_{[K]}})}\}$, since $[\Delta^*]_{\simeq_G} \in \mathcal{W}$ and $[\Delta^*]_{R_{[K]}} \in \mathcal{T}$. By Statement 2 and the fact that both $\Delta^* \in [\Delta^*]_{\simeq_G}$ and $\Delta^* \in [\Delta^*]_{R_{[K]}}$, $\Delta^* = \Sigma^*_{([\Delta^*]_{\simeq_G},[\Delta^*]_{R_{[K]}})}$. But by construction $\Delta \subseteq \Delta^*$, and consequently $Val(\psi, \mathfrak{A}^*, [\Delta^*]_{\simeq_G}, [\Delta^*]_{R_{[K]}}) = 1$, for every $\psi \in \Delta$, which proves the metatheorem. ■

Notes

[1] Examples of common nouns that are not considered to be sortal terms by many authors are words such as "thing" and "object". For a detailed discussion of these and other related topics see, for example, Geach(1980), Gallois(1988), Deutsch(2007), McGinn(2000), Noonan(1999) and Wiggins(2001). For a complete presentation of the different criteria proposed in the literature for a term to be considered a sortal and the issues regarding those criteria, see Grandy (2007).

[2] Sortal predication has been shown to be definable in terms of sortal quantification and sortal identity as follows: *x is an* $A =_{def} (\exists yA)(y =_A x)$. For this reason were are not including it in our definition of a language for sortals.

[3]For details on the modern version of conceptualism here assumed see, for example, Cocchiarella (2007)

[4] For the different discussions regarding relative identity, see Deutsch (2007). For arguments purporting to show that Leibniz law under sortal or relative identity is untenable, see Geach (1972, pp. 238-47),(1973) and (1980).

[5] Note that if $\gamma_0, \ldots, \gamma_n$ are wffs, then

(1) if $\odot(\gamma_0\& \ldots \& \; \gamma_n \;\&\; \exists M\varphi) \in \Gamma$ and D is a variable new to $\gamma_0, \ldots, \gamma_n$, $\exists M\varphi$, then by PL, $UG(s)$, A9, A4, T2, R$\boxdot$, Dist$\boxdot$ and definitions, $\odot\exists D(\gamma_0\& \ldots \& \; \gamma_n \;\&\; \varphi^D/M) \in \Gamma$. Since Γ is ω-complete, there is a sortal term variable L other than D which is free for D in φ^D/M such that $\odot(\gamma_0\& \ldots \& \; \gamma_n\& \; \exists D(D = L) \;\&\; \varphi^D/M^L/D) \in \Gamma$. Since D is new to $\exists M\varphi$, then $\varphi^D/M^L/D$ is φ^L/M

(2) If $\odot(\gamma_0\& \ldots \& \; \gamma_n \;\&\; \exists yS\varphi) \in \Gamma$ and z is a variable new to $\gamma_0, \ldots, \gamma_n$, $\exists yS\varphi$ then by UG, PL, A8, A3, T1, R$\boxdot$, Dist$\boxdot$ and definitions, $\odot\exists zS(\gamma_0\& \ldots \& \; \gamma_n \;\&\; \varphi^z/y) \in \Gamma$. But Γ is ω-complete and so there is a individual variable x other than z which is free for z in φ^z/y such that $\odot(\gamma_0\& \ldots \& \; \gamma_n \;\&\; \exists zS(z =_S x) \;\&\; \varphi^z/y^x/z) \in \Gamma$. Since z is new to $\exists yS\varphi$, then "$\varphi^z/y^x/z$" is "φ^x/y".

(3) (a) If $\odot((\gamma_0\& \ldots \& \; \gamma_n)\& \; \lfloor\varphi\rfloor) \in \Gamma$ and φ is of the form $\exists yS\sigma$, then by the ω-completeness of Γ there is a individual variable w other than y which is free for y in σ such that $\odot((\gamma_0\& \ldots \& \; \gamma_n)\&\lfloor \; \exists yS(y =_S w)\& \; \sigma^w/y)\rfloor) \in \Gamma$. It follows, by the irreflexivity of Γ, that there is an one-place predicate variable R which do not occur in $\odot((\gamma_0\& \ldots \& \; \gamma_n)\& \; \lfloor\exists yS(y =_S w)\&\sigma^w/y)\rfloor)$ and individual variable x such that $\odot((\gamma_0\& \ldots \& \; \gamma_n)\& \; \lfloor[K](Rx\&H\neg Rx) \;\&\; \exists yS(y =_S w) \;\&\; \sigma^w/y\rfloor) \in \Gamma$. (b) If $\odot((\gamma_0\& \ldots \& \; \gamma_n)\& \; \lfloor\varphi\rfloor) \in \Gamma$ and φ is not of the form $\exists yS\sigma$, then by the irreflexivity of Γ there is an one-place predicate variable R which do not occur in $\odot((\gamma_0\& \ldots \& \; \gamma_n)\& \; \lfloor\varphi\rfloor)$ and individual variable x such that $\odot((\gamma_0\& \ldots \& \; \gamma_n)\& \; \lfloor[K](Rx\&H\neg Rx)\& \; \varphi\rfloor) \in \Gamma$.

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Max A. Freund
Doctorado en Filosofía y Maestría en Ciencias Cognitivas
Universidad de Costa Rica
Apdo. 2627-2050 San Pedro de Montes de Oca - San José, Costa Rica
E-mail: max.freund@ucr.ac.cr