South American Journal of Logic
Vol. 3, n. 1, 2017
ISSN: 2446-6719

Is there a formula to express the *disparatae* medieval sentences? A positive answer

Juan Manuel Campos Benítez

Abstract

Our aim in this paper is to find a formula to express the *disparatae* medieval sentences as they are found in medieval octagons of opposition and equivalences. First we focus on the traditional square of opposition in order to stablish its rules of opposition. Then we take three different octagons and chose one as the *princeps analogatum*, which in turn take us to the *disparatae* square. Finally we propose a formula to describe the logical form of these sentences, which, however, is not exempt of problems.

1 The Traditional Square of Opposition

The square of opposition consists of four types of sentence, which show several relationships among them. The sentences can be universal or particular, affirmative or negative. Universal sentences cannot be simultaneously true but they can be simultaneously false; particular sentences can be simultaneously true but they cannot be simultaneously false. When universals are true, their particular sentences are also true but not conversely. When having the same terms, universal affirmative sentences and particular negative sentences cannot be simultaneously true or simultaneously false; the same holds for negative universal sentences and affirmative particular sentences.

Sentences having these relationships are known as contrary, subcontrary, subaltern and contradictory. We may as well express them in the following way: sentences in the same horizontal line are contraries or subcontraries; sentences in vertical lines are subalterns where the lower sentence is the subaltern to the upper; diagonal sentences are contradictories. Coloring the vertical (black), horizontal (blue and green) and diagonal (red) lines, the following square is given:[1]

[1] I follow suggestions by Béziau (2012), p. 11.

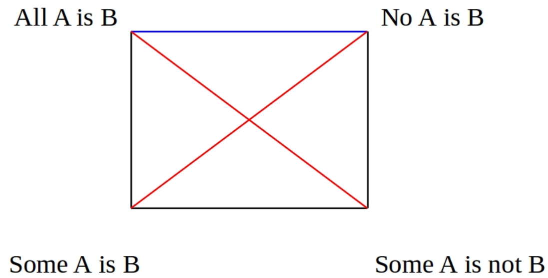

Figure 1:

1.1 Rules of the relations in the Square

These relationships follow the following rules:
Contraries: both cannot be true at the same time, but they can be both false.

Subcontraries: both cannot be false at the same time, but they can be both true.

Subalterns: if the universal sentence is true, the subaltern is also true, but not vice versa, i.e. the subaltern may be true and the universal false. Universals are "up" and particulars "down".

Contradictories: both cannot be true or false at the same time. If one is true the contradictory is false and vice versa.

There are formulas which express these relationships. To abbreviate I shall use the standard notation and letters A, E I, O so as to obtain:

Contraries: $\sim(A \wedge E)$, equivalent to $(\sim A \vee \sim E)$

Subcontraries: $(I \vee O)$, equivalent to $\sim(\sim I \wedge \sim O)$

Subalterns: $(A \rightarrow I)$, $(E \rightarrow O)$

Contradictories: $(A \leftrightarrow \sim O)$, $(E \leftrightarrow \sim I)$ equivalent to $\sim(A \leftrightarrow O)$, $\sim(E \leftrightarrow I)$

2 The Medieval Octagons

The medieval octagon can be depicted as a figure with eight types of sentence, each one having a Subject-Predicate form. Actually, there are three different octagons which can be found in John Buridan's *Summulae de Dialectica*, which were written in the XIV century.[2] The medieval octagons of opposition[3] are constructed by using an additional operator in a categorical sentence (which can be a modal operator, a quantifier of the predicate or a quantifier of a genitive relation such as x *belongs to* y). These situations

[2] I follow Gyula Klima's English translation. Cf. Buridan (2001).

[3] The reader may see the figures of the three octagons in Stephen Read (cf. Read, 2012, p. 100 and 106).

become more complex when the same relationships hold for sentences or type-sentences with two operators. Sentences will be contraries, subcontraries and subalterns according to each of the sentence's operators (simpliciter) or according to only one of them, either the subject or the predicate.

The subject in these octagons is always quantified. The predicate is explicitly quantified in the first octagon. The second octagon contains "modally" quantified propositions combining modality with quantification. The third octagon exhibits a quantified genitive relationship as a subject and an implicitly quantified predicate.

2.1 The explicitly quantified-predicate octagon

Here we have a quantified subject and a quantified predicate. For instance "Every A is every B". A subject-predicate sentence where both, S and P, are quantified terms, admits these combinations (the final "not" reflects Buridan's Latin usage), where A: Universal affirmative proposition, I: Particular affirmative proposition, E: Universal Negative proposition, O: Particular negative proposition.

	S P		S P
AA:	Every Every	AE:	Every Every not
AI:	Every Some	AO:	Every Some not
IA:	Some Every	IE:	Some Every not
II:	Some Some	IO:	Some Some not

Figure 2: Scheme 1

We shall employ these abbreviations since they summarize quite well the logical structure of the sentences and they will be helpful to understand the other octagons.

2.2 The modal octagon and the genitive octagon

The modal octagon shows a quantified subject and a modally quantified predicate. Possibility and necessity may be regarded as quantifiers as they show similarity to universal and particular quantifiers. They affect the verbal copula which in turn affects the predicate. This second octagon exhibits a rather complex predicate:

"Every A is possibly B" "Every A must be a B"

In the last example, "Every A must be a B", the presence of "a" indicates a hidden quantifier. If we take the mode 'necessary' as "true in every possible world" and the mode ‘possible’ as “true in at least one possible world”, we will have a second quantifier. The first quantifier applies to the subject A and the third to the predicate B, in these cases an implicit particular predicate for affirmative sentences; negative sentences contain a universal negative quantifier affecting their predicates, but they are not explicitly stated.

Now, we should notice that three operators make sixteen combinations, but we should remember that the first octagon has only S and P, but the modal octagon introduces a qualification of the predicate. The predicate is a rather complex predicate since it is a modalized one.

The third octagon has a complex subject: the genitive relationship between men and donkeys. The genitive sentence displays two quantifiers in the complex subject, one for the owner and one for the ownership; the third quantifier affects the predicate. The predicate is simple and indefinite in affirmative sentences, that is, it lacks a quantifier. For instance “Of every man every donkey runs” where the complex subject is “of every man every donkey” and “runs” or “is running” is the predicate, a particular predicate. Negative sentences are disttributed.

Modal schema: complex P

Affirmatives		Negatives	
S	**P**	**S**	**P**
A	AA	A	AE
A	AI	A	AO
A	IA	A	IE
A	I I	A	IO
I	AA	I	AE
I	AI	I	AO
I	IA	I	IE
I	I I	I	IO

Genitive schema: complex S

Affirmatives		Negatives	
S	**P**	**S**	**P**
AA	A	AA	E
AA	I	AA	O
AI	A	AI	E
AI	I	AI	O
IA	A	IA	E
IA	I	IA	O
II	A	II	E
II	I	II	O

Figure 3: Scheme 2

2.3 The analogy among the octagons

The first octagon is not as complex as the second and the third ones. Why does Buridan say, speaking of the two octagons, that “For in its own way this figure here is similar to the one there (...)”.[4] To understand the answer we should first know that Chapter 5

[4]Buridan (2001), Treatise 1, chap. 5, p. 43.

of his First Treatise, where the octagons appear, is about equipollences of propositions. If we have two quantifiers before the copula, as with the sentences we are treating here, those quantifiers are interchangeable with their equivalents; as a matter of fact Buridan presents nine equivalent sentences for each vertex of each octagon. Second, those sentences with two quantifiers follow the opposition rules, except in one "case" (to be treated in section 5). We already know the form of sentences with two quantifiers (Section 2.1).

Now, Buridan makes a kind of comparison among the three types of sentences:

	S P
"Every man some animal is"[5]	A – I
"Every man is able to run"	A – II
"Of every man some donkey is running"	AI – I

The third sentence's complex subject is made of a genitive ("of every man") and a nominative ("some donkey"), and the predicate. The second sentence shows a universal subject and a complex predicate where the first quantifier is of the I type. If we disregard the third quantifier of the modal sentence and the predicate of the genitive sentence, both sentences display an AI type sentence, just as in the first sentence. Buridan generalizes this strategy when he says:

> (...) if we match the oblique term before the nominative in the propositions with oblique terms, and the subject term of the propositions of unusual construction with the subject term of modal propositions, and, likewise, if we match the nominative term of the propositions with oblique terms, and the predicate of the propositions of unusual construction, with the mode of modal propositions. (Buridan 2008, p. 43).

If we do this to Schema 2 coloring the right-hand[6] sentences and observing the equivalences we obtain:

[5]This is a sentence of a usual construction (*de modo loquendi inconsueto*), ibid. p. 45.

[6]"Right", because we must be careful with negative sentences, which are distributives. For instance AAE is equivalent to AEI, AIE to AOI and so on.

Modal schema: complex P

Affirmatives S	Affirmatives P	Negatives S	Negatives P
A	AA	A	EI
A	AI	A	AO
A	IA	A	OI
A	II	A	IO
I	AA	I	EI
I	AI	I	AO
I	IA	I	OI
I	II	I	IO

Genitive schema: complex S

Affirmatives S	Affirmatives P	Negatives S	Negatives P
AA	A	AE	I
AA	I	AA	O
AI	A	AO	I
AI	I	AI	O
IA	A	IE	I
IA	I	IA	O
II	A	IO	I
II	I	II	O

Figure 4: Scheme 3

This happens to coincide exactly with our first schema; this gives us a reason to suspect that we have here the princeps analogatum to which the other octagons are secondary.

3 The relationships of the octagon

3.1 Contraries and subcontraries

Contraries: $\sim(AA \wedge AE)$, in the upper horizontal line; $\sim(AI \wedge AE)$, $\sim(AA \wedge AO)$, $\sim(AA \wedge IE)$, $\sim(AE \wedge IA)$ in diagonal lines.
Subcontraries: $(II \vee IO)$ in the lower horizontal line; $(II \vee IE)$, $(II \vee AO)$, $(IO \vee IA)$, $(IO \vee AI)$ in diagonal lines.
All these are shown in this figure

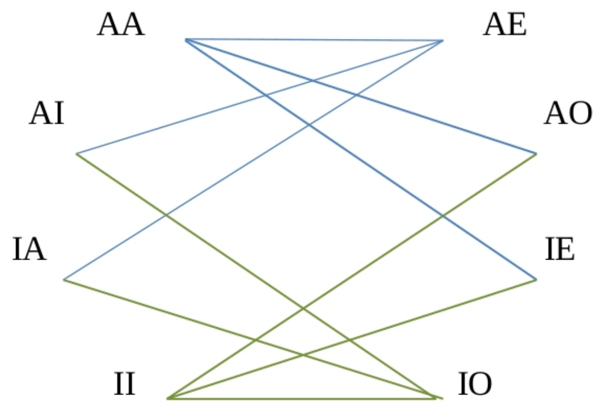

Figure 5:

3.2 Subalterns

Subalterns: $(AA \to AI)$, $(AA \to IA)$, $(AA \to II)$, $(AI \to II)$, $(IA \to II)$; $(AE \to AO)$, $(AE \to IE)$, $(AE \to IO)$, $(AO \to IO)$, $(IE \to IO)$
I will use arrows to express them:

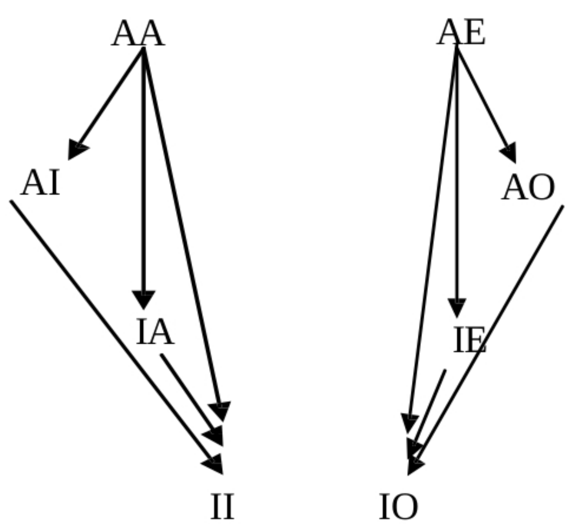

Figure 6:

3.3 Contradictories

Contradictories: $\sim(AA \leftrightarrow IO)$, $\sim(II \leftrightarrow AE)$; $\sim(AI \leftrightarrow IE)$, $\sim(IA \leftrightarrow AO)$ diagonal lines forming one outer square and one inner square. They form this spider-like figure.

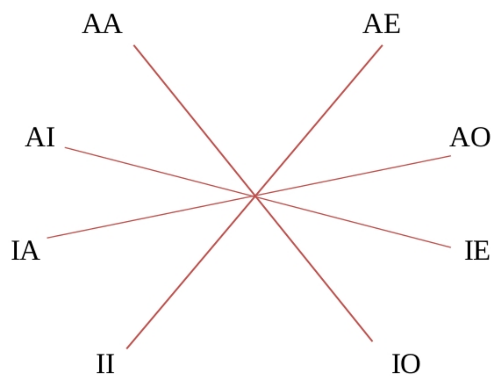

Figure 7:

Should we join all relationships, we obtain Figure 8, where every node of the octagon is joined to the others by some colored line.

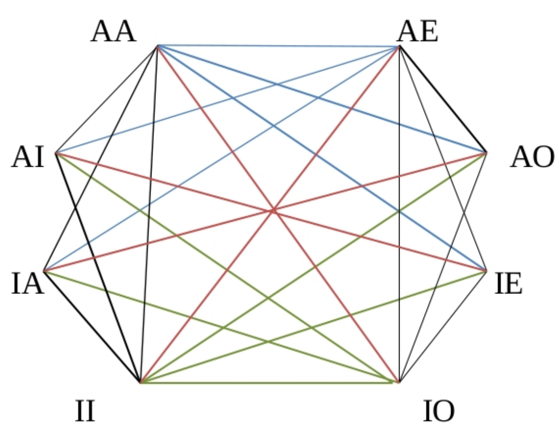

Figure 8:

4 The *disparatae* sentences

It is helpful to know about disparate terms in order to get a better understanding of disparate sentences. Let us take the two nouns, “man” and “stone”, which cannot both be said of Socrates. Indeed, if Socrates is a man Socrates is not a stone, and vice versa. Buridan says: “(...) terms are called ‘disparate’ because neither of them is opposed to, or follows from, the other” (p. 483).

Let us go back to the case discussed in Section 3.3. If we look carefully at Figure 8 above we notice that there are four nodes which are not linked by any coloured line. These are the nodes, joined by a purple line:

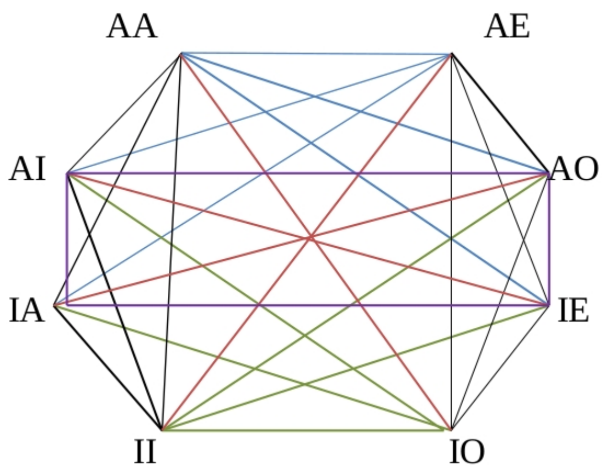

Figure 9:

We should notice that the four nodes (AI, AO, IA and IE) form an inner square inside the octagon. An outer square is formed by the nodes (AA, AE, II and IO). Both squares are squares of contradictories, but can also be seen, if we take the first and second line, as a square of universal sentences: AA, AE, AI and AO. Taking the third and fourth we obtain a square of particular sentences. We have lines (1, 2, 3, and 4) and columns (I and II) in the next section.

4.1 The disparate square

Lines 2 and 3 in column I have sentences with the same quality but different quantity (AI-IA) as well as lines 2 and 3 in column II (AO-IE). Column I and II on line 2 shows a pair of sentences with same quantity but different quality (AI-AO) and the same applies to line 3 column I and II (IA-IE). They form the disparate square.

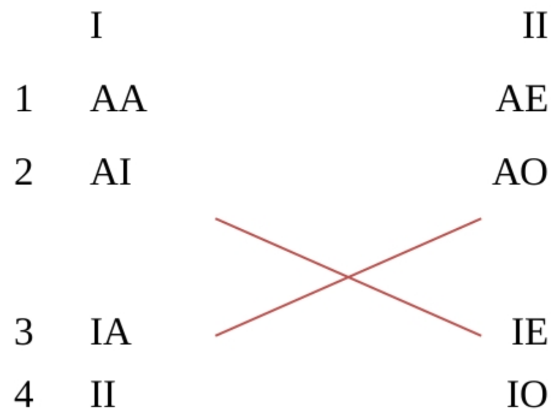

Figure 10:

The square may be seen as intermediate steps (AI-IA) between universal sentences (AA, AE) and particular sentences (II, IO). It also shows that two sentences with different quality (AI-AO) could be simultaneously true, for instance:

(AI-AO): "Every man is a runner" and "Every man is not some runner",
and the other pair (IA-IE) simultaneously false, for instance:

(IA-IE): "Some man is every runner" and "Some man is no runner".

Again, it also shows that two affirmative sentences (AI-IA) could be one true and the other false,

(AI-IA): "Every man is a runner" and "Some man is every runner",

and the same applies to their counterpart negative sentences:

(AO-IE): "Every man is not some runner" and "Some man is no runner".

(AI and IA) are neither subalterns nor contraries nor subcontraries nor contradictories to each other; the same holds for (AI-AO; IA-IE, AO-IE). Besides contradiction, no other relationship holds among these square of sentences. No wonder when Buridan (2001), p. 81, says:

> "And it appears to me that these are as it were disparate, obeying no law, neither the law of contradictories, nor the law of contraries, nor the law of subcontraries, nor that of subalterns, for such propositions can be true at the same time, because of their approaching subcontrariety on one part,

> and they can both be false at the same time, because of their approaching contrariety, on the other part; and since they are of diverse quality, while having the same subject and predicate, it is impossible that one should follow from the other, and since the one seems to share something from contradiction, the one can be true, while the other is false."

And a page later he adds:

> (...) just as if they had different subjects and different predicates (...).

5 A formula to express the *disparatae* sentences

The disparate square and its formulas are part of a bigger figure, the octagon. They are related to several formulas since they imply and are implied by others and are also contraries and subcontraries, and contradictories to others. However they are not related to each other and it seems strange were there no formula to express them.

Let us call disparate sentences D1, D2, D3 and D4; the relation among them Δ, which means they are not contradictories, nor subalterns, nor contraries nor subcontraries. All Ds are Δ-Related to just two of its neighbors, i.e. the next sentence in a vertical or horizontal line, but not through a diagonal line, as it is shown in this figure

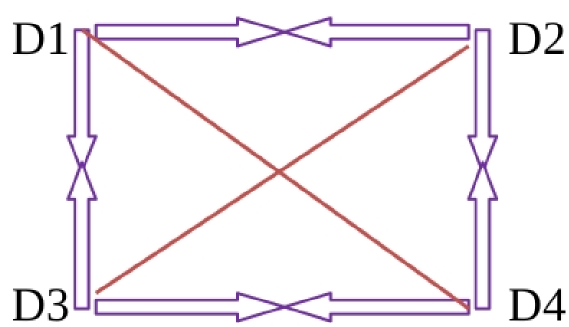

Figure 11:

So D1 is Δ-Related to D2 and D3 and D2 is Δ-Related to D1 and D4 and D3 is Δ-Related to D1 and D4 and D4 is Δ-Related to D3 and D2.
Now each D is -Related to two Ds but cannot be consistent with both, so we have

$$(D1 \wedge D2) \vee (D1 \wedge D3) \leftrightarrow D1 \wedge (D2 \vee D3) \leftrightarrow D1$$

$$(D2 \wedge D1) \vee (D2 \wedge D4) \leftrightarrow D2 \wedge (D1 \vee D4) \leftrightarrow D2$$
$$(D3 \wedge D1) \vee (D3 \wedge D4) \leftrightarrow D3 \wedge (D1 \vee D4) \leftrightarrow D3$$
$$(D4 \wedge D2) \vee (D4 \wedge D3) \leftrightarrow D4 \wedge (D3 \vee D2) \leftrightarrow D4$$

We cannot join these Ds conjunctively because it leads to a pair of contradictions:

$$(D1 \wedge D4) \wedge (D2 \wedge D3)$$

But joining them disjunctively, we obtain a disjunction of excluded middles:

$$(D1 \vee D4) \vee (D2 \vee D3)$$

Excluded middle because D1 and D4 as well as D2 and D3, are contradictory sentences, so we may rewrite the above formula as:

$$(D1 \vee {\sim}D1) \vee (D2 \vee {\sim}D2)$$

This turns out to be equivalent to:[7]

$$(D1 \rightarrow D2) \vee (D2 \rightarrow D1)$$

These two formulas, by the way, fit both sides of the description of "unconnectedness" given by Avi Sion (1996), chap. 6, 1:

> "Unconnectedness (or neutrality): two propositions are 'opposed' in this way, if neither formally implies the other, and they are not incompatible, and they are not exhaustive. Note that this definition does not exclude that unconnecteds may, under certain conditions, become connected (or remain unconnected under all conditions)."

Such is the formula that I propose to describe the logical behavior of disparate sentences. It seems that Excluded Middle is at the core of the octagon, but it is not free from possible "paradoxes".

[7]This seems to be odd, if *disparatae* sentences by definition do not imply each other. It looks like a kind of "paradox" (a paradox of relevance, for instance). These matters are out of the scope of this paper.

References

[1] J.-Y. Béziau. The New Rising of the Square of Opposition. In: J.-Y. Béziau and D. Jacquette (eds.), *Around and Beyond the Square of Opposition*, pp. 3–19. Basel: Springer, 2012.

[2] J. Buridan. *Summulae de Dialectica, An annotated translation with a philosophical introduction*. Gyula Klima, New Haven and London: Yale University Press, 2001.

[3] S. Read. John Buridan's Theory of Consequence and His Octagons of Opposition. In: J.-Y. Béziau and D. Jacquette (eds.), *Around and Beyond the Square of Opposition*, pp. 93–110. Basel: Springer, 2012.

[4] A. Sion. *Future Logic*, 1996. Avalaible at:
`http://www.thelogician.net/FUTURE-LOGIC/Actual-Oppositions-6.htm`

Juan Manuel Campos Benítez
Facultad de Filosofía y Letras
Benemérita Universidad Autónoma de Puebla (BUAP)
Av. Juan de Palafox y Mendoza No.229, Centro Histórico,
C.P 72000. Puebla, Pue, México.
E-mail: `juan.camposb@correo.buap.mx`