South American Journal of Logic
Vol. 7, n. 1, pp. 1–18, 2021
ISSN: 2446-6719

What is a Class? Carrollian Problems and Insights

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Abstract

Logician Lewis Carroll published in 1897 a logic of Classes in the symbolic tradition that was growing in his time. Through a comparison of the different editions of this work, this paper discusses some key difficulties that this logician faced in the shaping of his logic. We review consecutively problems and insights related to the formation of classes, the processes of Classification and Division, the relation between Classes and Individuals, the notions of Existence and Imaginariness, the Normal Form of Propositions, and finally the business of Logic.

Keywords: Class, Classification, Division, Dichotomy, Individual, Existence, Imaginariness, Relation.

1 Introduction

It is well-known that logician Lewis Carroll (1837-1898) worked for decades on a logic treatise, titled *Symbolic Logic*, that would make the subject accessible to a wide audience [37]. The work was expected to appear in three volumes, by level of difficulty. The first appeared in 1896 [13]. Subsequent volumes never appeared, although fragments have been posthumously published [6]. In recent decades, many scholars commented on Carroll's logical work, how it relates to his fictional writings, its logical merits and how it stands within the logic of its time [1; 2; 3; 22; 23; 39; 42].

Carroll's logic book does not have the erudition that characterised many of the logic treatises of his time. A comparison with John Venn's book of the same title, *Symbolic Logic*, whose second edition appeared in 1894 is insightful [53]. Venn thoroughly discusses the works of his predecessors and contemporaries; he assesses existing views and justifies those he held. There really is nothing of the sort in Carroll's work: he rather straightforwardly delivers his doctrine through a set of definitions and rules that readers are invited to use

in order to solve a specific set of problems.[1] While Venn mainly argues, Carroll merely exposes. An exception to this is the short appendix addressed to Teachers where Carroll discusses some of his views. But the learner, to whom the book is primarily addressed, is dispensed of it. One simply does not find in Carroll's logic anything equivalent to Venn's lengthy, dense and insightful discussions on the choice of a symbolic language, on intensive interpretations, on propositional logic, etc. Even on subjects that both authors address, the difference is straightforward. For instance, Venn devotes an 11-page chapter to the concept of a Universe of discourse [53, pp. 245-255], against two lines in Carroll's treatise [16, p. 12].

Of course, Carroll's book has its own merits, notably its rich pedagogical apparatus (examples, diagrams, tables) and its wonderful set of examples, with which Venn cannot compete. But that is precisely the point: While Venn's book appears as a serious scholarly treatise on the subject, Carroll rather appears to the reader as an elegant elementary manual. The reason why Carroll's work resembles a practical manual is that it precisely is what Carroll intended it to be. Indeed, he explained in his preface that he "carefully avoided all difficulties which seemed to [him] to be beyond the grasp of an intelligent child of (say) twelve or fourteen years of age" [16, p. xvi]. After enumerating the practical skills that one is expected to acquire, Carroll addresses the reader: "*Try it.* That is all I ask of you!" [16, p. xvii]. The book is written in such a way as to guide the reader through a set of rules to conduct certain procedures, such as reducing propositions to normal forms, representing propositions with diagrams or symbols, inferring conclusions, interpreting marked diagrams and symbolic formulas, detecting fallacies, etc.

Carroll's style made his manual highly adequate for beginners but leaves the scholar frustrated. Indeed, it says little on the development of his ideas, his philosophical motivations or his familiarity with the logical works of his time. To address these questions, the historian has to rely on indirect sources, such as his correspondence, his private diaries and the catalogue of his private library [34; 38; 45]. Another source that has so far been overlooked is the modifications that Carroll introduced in the three subsequent editions of his book between 1896 and 1897 [14; 15; 16].[2] Although many are minor alterations, most reveal

[1] Such problems generally consisted in finding the conclusion that follows from a set of propositions offered as premises. Syllogisms may be seen as a special case of this general problem. The elimination of undesired terms from premises produces the conclusion through a calculus which became prominent with the development of the algebra of logic [27]. As such, Carroll may be said to belong to the 'research program' that was centred on Calculus, unlike other logicians who worked on the Analysis of logic [7].

[2] There is an exception here: a note inserted by Carroll in the second edition of his *Symbolic Logic* was noticed by Carrollian scholars, for reasons not directly related to his logic work, however. In this note, Carroll denied a story according to which he presented certain

deep difficulties that Carroll faced in the shaping of his logical theory. In this paper, we consider specifically the issues related to Carroll's conception of Classes.

2 The formation of Classes

Carroll's *Symbolic Logic* opens with a set of statements on Things and their Attributes. We are told that "The Universe contains 'Things.' [...] Things have 'Attributes.' [...] One Thing may have many Attributes; and one Attribute may belong to many Things" [13, p. 1]. None of the concepts 'Universe', 'Thing' and 'Attribute' is strictly defined but the reader is offered examples to make their meaning understandable.[3] Classes are the first objects that are defined by Carroll, albeit indirectly. This is achieved through the description of two mental processes through which classes are formed: Classification and Division. Both processes produce a 'Set' or a 'Group' of Things, which Carroll eventually called a Class. Throughout the revision of his work, Carroll made some interesting changes in his definitions of these processes.

Classification is defined in the first edition as "a Mental Process, in which we imagine that we have picked out, from a certain Set of Things, all that possess a certain Attribute (or Set of Attributes), and have put them together in a group by themselves. Such a group is called a 'Class"' [13, p. 2]. However, starting from the second edition, this definition is slightly altered, for we are told that Classification is "a Mental Process, in which we imagine that we have put together, in a group, certain Things. Such a group is called a 'Class"' [14, p. 2; 16, p. 1 1/2]. Unlike the earlier definition which mentions picking out Things from a certain Set before putting them together, Carroll's new definition does not. The reason for this alteration is that Carroll discovered a peculiar Class, the Universe, whose process of formation does not involve this Picking out phase.

Indeed, Carroll's new definition is followed by an enumeration of three ways in which the process of classification can be performed: (1) We put together all Things to form the Class that contains the whole Universe. (2) We pick out from the Universe and put together Things that have a certain Attribute. (3) We pick out from a certain Class all its members that have a certain Attribute [14, p. 2]. We clearly see that the first case is formulated differently because it serves for the formation of the whole Universe. Hence, we cannot

mathematical books to Queen Victoria. This note was dropped in the fourth edition. For a discussion of this story, see [40].

[3]Later, Carroll added the term 'Adjunct' which stands for any Attribute or set of Attributes. It is introduced to "avoid the constant repetition of the phrase "Attribute or Set of Attributes"" [16, p. 1].

pick out Things from a mother class that would be larger or equivalent to the Class we wish to form (the Universe). In the other cases, we pick out Things from another Class that may be the Universe (in the second case) or a certain Class (in the third class). These three methods actually need to be performed in that order: The first forms the Universe, the second forms certain Classes from the Universe, and the third forms sub-Classes from certain Classes. Although this new definition differentiates the formation of the Universe from the formation of other Classes, it allows Carroll to make the Universe as a Class itself, which is the class of all Things. Carroll reported this change in his preface to the second edition: "I have adopted a new definition of the Process, which enables me to regard the whole Universe as a 'Class,' and thus to dispense with the very awkward phrase 'a Set of Things"' [14, p. ix].

Carroll's second method for the formation of Classes is Division. It consists in dividing a Class into sub-Classes. This process was well-known and widely discussed among Carroll's contemporaries [25, pp. 59-68; 28, pp. 692-698; 29, pp. 89-104; 52, pp. 309-342; 56, pp. 139-169]. John Neville Keynes devoted a long appendix of his *Formal Logic* to 'the doctrine of division' [30, pp. 441-449]. He first reviewed several types of it: physical (e.g., dividing a chemical object into its constitutive elements), metaphysical (enumerating the characteristics of an object), verbal (distinguishing the different meanings of a word), etc. Then, Keynes identified the two main principles that rule logical Divisions: (1) Sub-Classes should be mutually exclusive so that no individual belongs to more than one sub-Class. (2) Sub-Classes should exhaust the mother-Class, so that every individual of the mother-Class belongs to a sub-Class. Dichotomy is the simplest instance of a logical Division. It consists in dividing a certain Class x into two complementary sub-Classes $x\ a$ and $x\ non-a$. This process follows Keynes' two rules in virtue of the principles of non-contradiction and the excluded-middle respectively.[4]

[4] An advantage of Dichotomy is that all other Divisions can be reduced to it. Suppose we are given a Division of Class x into three sub-Classes a, b, and c which are mutually exclusive and exhaust x. It is possible to obtain this Division by a series of Dichotomies, starting with x and dividing at each step existing Classes into two sub-Classes depending on how they relate to a, b, and c successively. We eventually obtain eight sub-Classes: (1) $a\ b\ c$, (2) $a\ b\ non-c$, (3) $a\ non-b\ c$, (4) $a\ non-b\ non-c$, (5) $non-a\ b\ c$, (6) $non-a\ b\ non-c$, (7) $non-a\ non-b\ c$, (8) $non-a\ non-b\ non-c$. Finally, we declare empty sub-classes (1), (2), (3), (5) and (8). This leaves sub-Classes (4), (6) and (7) which correspond to the desired sub-Classes a, b and c respectively. This technique, based on Dichotomy, was central to the new logic developed by George Boole, William Stanley Jevons and Venn. Boole famously identified the law $x^2 = x$ as the fundamental law of thought and argued that it was "a consequence of the fact that the fundamental equation of thought is of the second degree, that we perform the operation of analysis and classification, by division into pairs of opposites, or, as it is technically said, by *dichotomy*" [9, p. 50]. The logical process of Dichotomy was naturally familiar to Boole's followers who acknowledged its central role in the design of their symbolic

In the first edition of his treatise, Carroll defined Division as "a Mental Process, in which we think of a certain Set of Things, and imagine that we have divided it into two or more smaller Classes" [13, p. 4]. This definition did not change in subsequent editions, except for the substitution of 'Class of Things' for 'Set of Things' [16, p. 3]. This definition does not explicitly make Division comply to Keynes' two principles. When a Division complies to Keynes second principle, Carroll called it an 'Exhaustive Division'. Carroll mentioned this type of division in his early booklet *The Game of Logic* (1887), where he writes that ""nice" and "not-nice" make what we call an '*exhaustive*' division of the class "new Cakes": i.e., between them, they *exhaust* the whole class, so that all the new cakes, that exist, must be found in one or the other of them" [10, p. 8]. Apparently, Carroll did not use a specific name for Divisions that comply to Keynes' first principle. Rather, he had coined name for those that do not. Such Divisions where classes are not (all) mutually exclusive, are called Cross-Divisions. Both types of Division (Exhaustive Division and Cross-Division) are mentioned in an early table of contents of *Symbolic Logic*, printed around 1894 [3, p. 77].

Carroll's peculiar conception of Division impacted his treatment of Dichotomy. Indeed, in the three first editions of his treatise, Dichotomy was merely defined as "Division into *two* classes" [13, p. 4], without requiring exhaustivity and mutual exclusion between sub-Classes. When these requirements are enforced, one obtains a specific type of Dichotomy that Carroll named 'Dichotomy by contradiction' which he defines as follows: "If a certain Set of Things be divided into two classes, one of which contains all the Things possessing a certain Attribute (or Set of Attributes), and the other all the Things *not* possessing it (that is, all the rest of the Set), the Process is called 'Dichotomy by Contradiction."' [13, p. 4]. However, in his fourth edition, Carroll abandoned his distinction between 'Dichotomy' and 'Dichotomy by contradiction' and simply used the former term in the sense of the latter, as other contemporary logicians did:

> "If we think of a certain Class, and imagine that we have picked out from it a certain smaller Class, it is evident that the *Remainder* of the large Class does *not* possess the Differentia of that smaller Class. Hence it may be regarded as *another* smaller Class, whose Differentia may be formed, from that of the Class first picked out, by prefixing the word "not"; and we may imagine that we have

logic, notably Jevons logical alphabet [28, pp. 89-96] and Venn's compartmental logic [53, p. 111]. The idea is to divide the universe by Dichotomy, depending on the number of terms involved in an argument, then to determine the state of the resulting sub-classes regarding their emptiness or occupation [20]. Carroll also used this very process in the construction of his diagrams [36].

> *divided* the Class first thought of into *two* smaller Classes, whose Differentiae are *contradictory*. This kind of Division is called 'Dichotomy"' [16, p. 3 1/2]

The formal nature of Dichotomy raises the question of those "*two* smaller Classes, whose Differentiae are *contradictory*", as Carroll described them. It is common to refer to the "Class first picked out" as the positive class, and the other as the negative one. The indefinite nature of the latter class led Keynes to undermine its importance from a material viewpoint [30, p. 446]. However, from a formal standpoint, many logicians, including George Boole and Augustus De Morgan, viewed the two classes formed by Dichotomy on the same footing, and argued that their signs were merely relative to each other [19, pp. 1-3; 26, p. 16]. Venn held a similar position but did not reflect this view in the design of his diagrams [53, pp. 249-250]. Carroll also defended the equal status of complementary classes and ridiculed the "morbid fear" of traditional logicians who act like "frightened children" when they face propositions that contain negative attributes [16, p. 172] (see [24]). Carroll concluded that:

> "Under the influence of this unreasoning terror, they plead that, in Dichotomy by Contradiction, the *negative* part is too large to deal with, so that it is better to regard each Thing as either included in, or excluded from, the *positive* part. I see no force in this plea: and the facts often go the other way [...] For the purposes of Symbolic Logic, it is so *much* the most convenient plan to regard the two sub-divisions, produced by Dichotomy, on the *same* footing, and to say, of any Thing, either that it "is" in the one, or that it "is" in the other, that I do not think any Reader of this book is likely to demur to my adopting that course." [16, p. 172]

For instance, instead of affirming that a certain x is-not y, it is more convenient to state that a certain x is $not-y$, hence moving the negation from the copula to the attribute. This technique will prove decisive in the shaping of Carroll's typology of propositions. He also took care and pride in reflecting the equal status of opposite classes in the design of his diagrams [8].

As Carroll used both Classification and Division to form Classes, one might wonder to what extent the two processes are connected since both consist in separating the Things having a certain Attribute or set of Attributes from those that do not have it. Carroll's contemporary E. E. Constance Jones summarised well the interplay between the two processes:

> "It may be said that Division and Classification are the same thing looked at from different points of view; any table presenting a Division presents also a Classification. A Division starts with unity,

> and differentiates it; a Classification starts with multiplicity, and reduce it to unity, or, at least, to order." [29, p. 101].

However, Carroll's hesitations bring to light an essential difference: Classification alone makes it possible to form the Universe.

3 Classes and individuals

Carroll insisted on the mental nature of Classification, and hence argued that "we may perform it without knowing whether there *are*, or *are not*, any existing Things which are Members of it. If there *are*, the Class is said to be 'Real; if *not*, it is said to be 'Imaginary."' [13, p. 2]. To illustrate these two sorts of Classes, Carroll states that the Class 'English Towns having four million inhabitants' was Real, while the Class 'English Towns having ten million inhabitants' was Imaginary [13, p. 2]. Carroll did not precisely tell what Reality and Imaginariness stand for but, in a later edition, he stated that to assert the Reality of a Class is to assert its "*real* existence" [16, p. 11], a wording that suggests that Imaginariness stands for some kind of 'Imaginary existence'. This does not easily match with the definition above where we were told that Imaginary Classes do not contain Existing Things.

What makes the situation even more complicated is that Carroll apparently distinguished the Class of 'Things' which stands for the whole Universe [16, p. 1 1/2] from that of 'Existing Things' [16, p. 11], which suggests the existence of a Class of 'Non-Existing Things'. But in what sense could a Class of Non-existing Things exist? Carroll's commentary on the notion of Existence is of little help here:

> "By "existence" I mean of course whatever kind of existence suits its nature. The two Propositions, "*dreams exist*" and "*drums exist*", denote two totally different kinds of "existence". A *dream* is an aggregate of ideas, and exists only in the *mind* of a *dreamer*: whereas a *drum* is an aggregate of food and parchment, and exists in the *hands of a drummer*." [16, p. 166]

In this passage, we learn that different types of Existence exist, as Existing Things may be mental or material, for instance. But 'what kind of existence suits' the nature of Non-existing (Imaginary) Things? Carroll's treatment suggests a kind of Imaginary Existence that anticipates the views that will be promoted by logician Hugh MacColl in subsequent years.[5]

[5]MacColl was familiar with Carroll's work, since he reviewed the book in *The Athenaeum* [32]. This reading apparently encouraged him to reinvest the logic scene after a long inter-

Let us first remind that Carroll, like many of his contemporaries, acknowledged that we commonly restrict the scope of our discourse to a specific part of the Universe [18]. De Morgan explicitly introduced the idea [19, p. 2], before the concept became known as the 'Universe of Discourse', an expression that was apparently coined by Boole [9, p. 42]. Subsequently, Venn argued that "the conception of a universe is seen to be strictly speaking extra-logical; it is entirely a question of the application of our formulae, not of their symbolic statement" [53, p. 250].[6] Carroll made a thorough use of the concept of a Universe of Discourse which he defined as: "The Genus, of which the two Terms [of a Proposition] are Specieses, is called the 'Universe of Discourse,' or (more briefly) the 'Univ."' [16, p. 12]. Consider, for instance, the proposition: "No one takes in the *Times*, unless he is well-educated". Whoever enunciates this statement connects two classes: "Persons taking in the Times" and "persons who are well-educated". These terms are Speciese of the Genus "Persons" which is chosen as the Universe of Discourse of the Proposition. Indeed, no reader is expected to believe that the claim was made about anything other than "Persons" [16, p. 15].

A difficulty that led to some controversy among Carroll's contemporaries and immediate followers concerned the extent to which individuals are held to exist in the Universe of Discourse. MacColl argued that a Universe of Discourse is constituted of two sub-universes, one for realities and one for unrealities:

> "Let e_1, e_2, e_3, etc. (up to any number of individuals mentioned in out argument or investigation) denote the universe of *real existences*. Let $0_1, 0_2, 0_3$, etc., denote our universe of *non-existences*, that is to say, of unrealities, such as *centaurs*, *nectar*, *ambrosia*, *fairies*, with self-contradictions, such as *round squares*, *square circles*, *flat spheres*, etc., including, I fear, the non-Euclidean Geometry of four dimensions and other hyper-spatial geometries. Finally, let *S1*, *S2*, *S3*, etc., denote our *Symbolic Universe*, or "Universe of Discourse," composed of all things real or unreal that are named or expressed by words or other symbols in our argument or in-

ruption during which he mainly produced works of fiction [4; 5]. It is however unlikely that MacColl and Carroll ever met or privately communicated. In the mid-1890s, Carroll started a large controversy on the nature of hypotheticals which led him to a correspondence with many of his British colleagues but MacColl apparently was not one of them [35]. Both logicians were to some extent outsiders in the British logical community and within the growing modern logic tradition [41].

[6] Consequently, Venn did not include a representation of the Universe in his diagrams, unlike most of his immediate followers Allan Marquand, Alexander Macfarlane and Lewis Carroll [8]. This choice heavily impacted on the construction of diagrams for a high number of terms since rectangular diagrams, which enclose the universe, performed better. Venn himself used Marquand diagrams in the second edition of his *Symbolic Logic* [43; 44].

> vestigation. By this definition we assume our Symbolic Universe (or "Universe of discourse") to consist of our Universe of realities, e_1, e_2, e_3, etc., together with our universe of unrealities, $0_1, 0_2, 0_3$, etc., *when both these enter into our argument*" [33, p. 74].

MacColl's views faced strong resistance from both traditional and modern logicians. For instance, A. Wolf argued in 1905 that there was no existence outside the universe of reality:

> "We hear of things existing in some universe of discourse even if they do not exist in the universe of reality. Thus, for instance, we are told that centaurs, the gods of the Greeks, the Muses, etc. do not exist in the world of reality, but that they do exist in the universe of mythology, of fiction, etc., as the case may be [...] There is but one universe of reality, and what does not exist in it exists nowhere else. That something may exist in some universe of discourse even if it does nto exist in the universe of reality is an extravagant conception, and does not really mean what it seems to mean" [57, pp. 67-71].

Bertrand Russell also contended that MacColl confused two types of Existences. On the one hand, Existence, as used in philosophy and common life, tells if an individual is found in Reality. In this sense, Socrates is said to exist while Hamlet is not. On the other hand, Existence, as used in mathematics and symbolic logic, consists in affirming whether a Class has or has not individuals [49, p. 398]. Hence, one does not need to distinguish two universes within the Symbolic Universe because any individual in that Universe is taken to exist by the logician, regardless of its existence or non-existence in Reality [49, p. 399]. In this manner, Russell distinguishes the existence of individuals from that of Classes and considers the former as an extra-logical issue. The logician is merely concerned with existence of Classes. For him, "there are no unreal individuals; so that the null-Class is the class containing no members, not the class containing as members all unreal individuals" [50, p. 491] (see [48]).

Carroll seems closer to MacColl than to Russell. It is true that he speaks of the Existence and Imaginariness of Classes, but this conception is closely related to the Existence and Imaginariness of individuals, for an Imaginary Class is not necessary null. All we are told is that it contains no existing individual, but it may well contain imaginary individuals. What makes things more complicated is that Carroll, like most British logicians of his time, did not strictly distinguish Classes and individuals. This is seen in his treatment of individuals classes, *i.e.*, Classes containing a single member. Carroll stated

that "any single Thing, which we can name so as to distinguish it from all other Things, may be regarded as a one-Member-Class" [16, p. 2 1/2]. This view entailed difficulties in his formulation of singular propositions, as can be seen through the changes he made in his successive editions.

Carroll treated singular propositions as universals. Hence, they ought to be translated into the normal form of relation "All x are y". In his treatise, Carroll considered, for illustration, the singular proposition "John is not well". In the first edition, he stated that the subject of this proposition was "the Class of men who are here referred to by the name "John"" [13, p. 17]. Consequently, the proposition was translated into the normal form: "All Johns are men who are not well" [13, p. 17]. George Osborn criticised this statement in a private correspondence with Carroll and argued that it ought to be rewritten to make clear that the subject of the proposition was a "one-Member Class" [12]. Carroll modified his text accordingly in the second edition where we are told that the subject of that singular proposition was "the one-Member Class "Johns," i.e., the Class of men who are here referred to by the name "John," of which Class only *one* specimen exists" [14, p. 16]. The normal form itself became: "All Johns are not-well" [14, p. 16]. This expression was kept in the third edition [15, p. 12], but it got again criticised by MacColl:

> "Lewis Carroll paraphrases the simple proposition "John is not well" into the astounding assertion that *all Johns are men who are not well*, as if the illness of one member of that numerous and widely scattered family must necessarily involve the illness of all! Among the many Johns of Lewis Carroll's acquaintance is there really *not one* who enjoys good health?" [32, p. 520]

Carroll changed again his explanation in the fourth edition of the treatise, where we are told:

> "Let us take, as an example, the Proposition "John is not well". This of course implies that there is an *Individual*, to whom the speaker refers when he mentions "John", and whom the listener *knows* to be referred to. Hence the Class "men referred to by the speaker when he mentions 'John"' is a one-Member Class, and the Proposition is equivalent to "All the men, who are referred to by the speaker when he mentions 'John', are not well."" [16, p. 10].

4 Existence and relations of Classes

Throughout his four editions, Carroll maintained a distinction between two forms of propositions: 'Propositions of Existence' and 'Propositions of Re-

lation' [16, pp. 11-12]. A proposition of Existence asserts the Existence or Imaginariness of a certain Class. It can be of two forms: I ("Some x exist") or E ("No x exist"). A proposition of relation rather asserts a relation between two Classes. It can be of three forms: I ("Some x are y"), E ("No x are y") or A ("All x are y"). Carroll considered the latter form "All x are y" as a double proposition in that it is equivalent to the combination of propositions "No x are *non-y*" and "Some x are y" [16, p. 18].[7] As such, Carroll preserved the traditional substitution of I to A propositions, in contrast to other symbolic logicians. However, Carroll did not include traditional propositions of the O form: "Some x are not y". Indeed, he viewed these as equivalent to "Some x are *non-y*" which are of the I form [16, pp. 171-172].

It is important to insist on the fact that, although Carroll distinguishes Things and their Attributes, Propositions in their normal forms pertain to Classes as collections of Things alone. This extensional interpretation, similar to that of most symbolic logicians, prevented some difficulties which Carroll had noticed. For instance, one may wonder how the copula ought to be interpreted in a proposition of Relation. For Carroll, a proposition of the form "All x are y" should implicitly be read as "All ax are ay", where a stands for the Universe of Discourse. Hence, the predication of an Attribute (y) to a Thing (x), which a proposition intends, is reduced in the normal form to the inclusion of a Class of Things (ax) into another Class of Things (ay). Carroll explained this view ten years earlier in *The Game of Logic*:

> "[I]f you put "is" or "are" between the name of a Thing and the name of an Attribute (for example "some Pigs are pink") you do not make good sense (for how can a Thing be an Attribute?) unless you have an understanding with the person to whom you are speaking. And the simplest understanding would, I think, be this – that the substantive shall be supposed to be repeated at the end of the sentence, so that the sentence, if written out in full, would be "some Pigs are pink (Pigs)". And now the word "are" makes quite good sense" [10, pp. 2-3].

For the purposes of his symbolic and diagrammatic notations, Carroll is led to transform all propositions into forms of Existence to ease their expression. For instance, a proposition of the form "Some x are y" becomes "Some xy exist" while a proposition of the form "No x are y" becomes "No xy exist".

[7] In other places of his treatise, Carroll identified these two subaltern propositions as "No x is *non-y*" and "Some x exist". Carroll explained that "the Proposition "Some xy exist" contains *superfluous information.* "Some x exist" is enough for our purpose" [16, p. 72]. Indeed, given that "No x is *non-y*", it follows that if any x exists, x must be y since y and *non-y* are complementary (See [21, pp. 40-41]).

The reduction of propositions to forms of Existence was known to Carroll's contemporaries. Several authors observed that it was already found in Franz C. Berntano's *Psychologie vom Empiricischen Standpunkte* (1874) [17, p. 264; 51, p. 63]. Brentano's logic work was known to British logicians, notably through J. P. N. Land's account of it in the journal *Mind* in 1876 where he reported that: "the main feature of [Brentano's] reconstruction of logical doctrine consists in reducing all categorical propositions to what he calls existential propositions doing away with the familiar distinction between subject and predicate terms" [31, p. 289]. Venn rightly observed that this innovation was already implicit in some versions of Boole's logic notation [53, pp. 184-185]. Interestingly, Carroll himself used such algebraical notation as early as 1876 [54, pp. 463-464].

The transformation of propositions of Existence into propositions of Relation proves more complicated. Indeed, while the former pertains to a single Class, the latter asserts a relation between two Classes. Carroll stated that the missing Class ought to be "Existing Things" but hesitated as to whether it should stand for the Subject or the Predicate of the resulting proposition of Relation. Let there be a proposition of Existence "Some x exist" and it is desired to translate it into a form of Relation. The instructions of the first edition would translate this proposition into the form of Relation: "Some x are Existing things" [13, p. 11]. But the fourth edition would rather lead to the form of Relation "Some Existing things are x" [16, p. 11]. This change was introduced in the second edition and reported by Carroll in its preface:

> "I have adopted a new 'normal form,' in which the Class, whose existence is affirmed or denied, is regarded as the *Predicate*, instead of the *Subject*, of the Proposition, thus evading a very subtle difficulty which besets the other form. These subtle difficulties seem to lie at the roots of every Tree of Knowledge, and they are *far* more hopeless to grapple with than any that occur in its higher branches [...] And, in the present work, the difficulties of the "5 Liars" Problem [...] are "trifles, light as air," compared with the bewildering question "What is a Thing?"" [14, p. ix].

This passage gives an idea of the seriousness of the problem that Carroll faced but does not tell precisely what that difficulty might be. What motivated Carroll's change from the normal form "Some x are Existing things" to "Some Existing things are x"? According to Arthur N. Prior, the first form suggests that Class x may be divided into two sub-Classes: that of xs which exist and that of xs which do not exist [47, p. 310].

Interestingly, MacColl pointed out earlier a paradoxical result of the same nature while discussing Carroll's theory of existential import. It is reminded that Carroll chose to maintain the existential import of universal affirmative

propositions [16, p. 19]. In his Appendix to Teachers, Carroll justified this choice by its conformity to the "accepted facts of Logic" and the "actual facts of life" [16, p. 167]. Despite some hesitations, his opinion on this subject did not change since at least 1884 [55, p. 156]. In *The Game of Logic* (1887), he expressed his view as follows:

> "[I]n every Proposition beginning with "some" or "all", the *actual existence* of the 'Subject' is asserted. If, for instance, I say "all misers are selfish," I mean that misers *actually exist*. If I wished to avoid making this assertion, and merely to state the *law* that miserliness necessarily involves selfishness, I should say "no misers are unselfish" which does not assert that any misers exist at all, but merely that, if any *did*, they *would* be selfish" [10, p. 19].

The same position was maintained in the four editions of *Symbolic Logic,* ten years later. It has been suggested that Carroll may have planned to change his theory of Existential import in subsequent volumes [1]. It is true that Carroll inserted a note in the Existential import section of his fourth edition stating that "the rules, here laid down, are *arbitrary*, and only apply to Part I of my "Symbolic Logic." [16, p. 19]. However, it is unknown what changes he might have made.

Carroll's view corresponded to what is found in traditional logic treatises of his time [46]. However, it was different from that of most symbolic logicians who dropped the Existential import of universal affirmatives to ease their notations and calculations. Carroll's position was severely criticised by MacColl:

> "Hence, according to Lewis Carroll's ruling, the assertion "All S is P," if correct, implies that S really exists. Now there is a certain theorem, generally (we will not rashly say universally) accepted as valid, which does not seem to accept this ruling with the meekness that it ought. The theorem is that "A is A." It will generally be admitted, we think, that "All non-existent things are non-existent"; yet, according to the author, this proposition would imply that non-existent things really exist: a rather staggering assertion in the prosaic world of our experience, though the most fundamental of all axioms in Wonderland" [32, p. 520].

5 Conclusion

Despites his difficulties with the forms of Existence and Relation, Carroll maintained them throughout his four editions where they play a central role in his

logical theory. However, it must be said that this distinction appeared rather late his investigations, even though it is implicit in some of his early notations. Two preliminary tables, presumably printed in 1894, give an interesting insight on the contents of the projected treatise.[8] In the earlier table, kinds of propositions are mainly defined by the number of terms involved in them. Hence one meets with Binomial, Trinomial and Polynomial Propositions. Interestingly, a handwritten correction introduced a new typology: "Propositions are of 2 kinds: (1) asserts existence, or non-existence, of a single Class. (2) Asserts relation of one Class to another, as to being excluded from it or included in it." [11, p. 1]. The second table (reprinted in [3, pp. 75-88]) incorporated the handwritten corrections made on the earlier table and revealed a new section devoted to "*Propositions as to Existence, and as to Relation*" [3, p. 77]. Interestingly, this addition also affected Carroll's idea of the business of logic. Indeed, in the early table, we are told that "Logic deals with the relations of Classes, with regard to their including, or excluding, one another" [11, p. 1]. But in the second table, this entry is updated, and we are presently told that "Logic deals with the existence of Classes, and with their relations to one another" [3, p. 77]. Unfortunately, this entry as well as the entire opening chapter on the definition of logic did not appear in the first edition of the book published two years later.

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